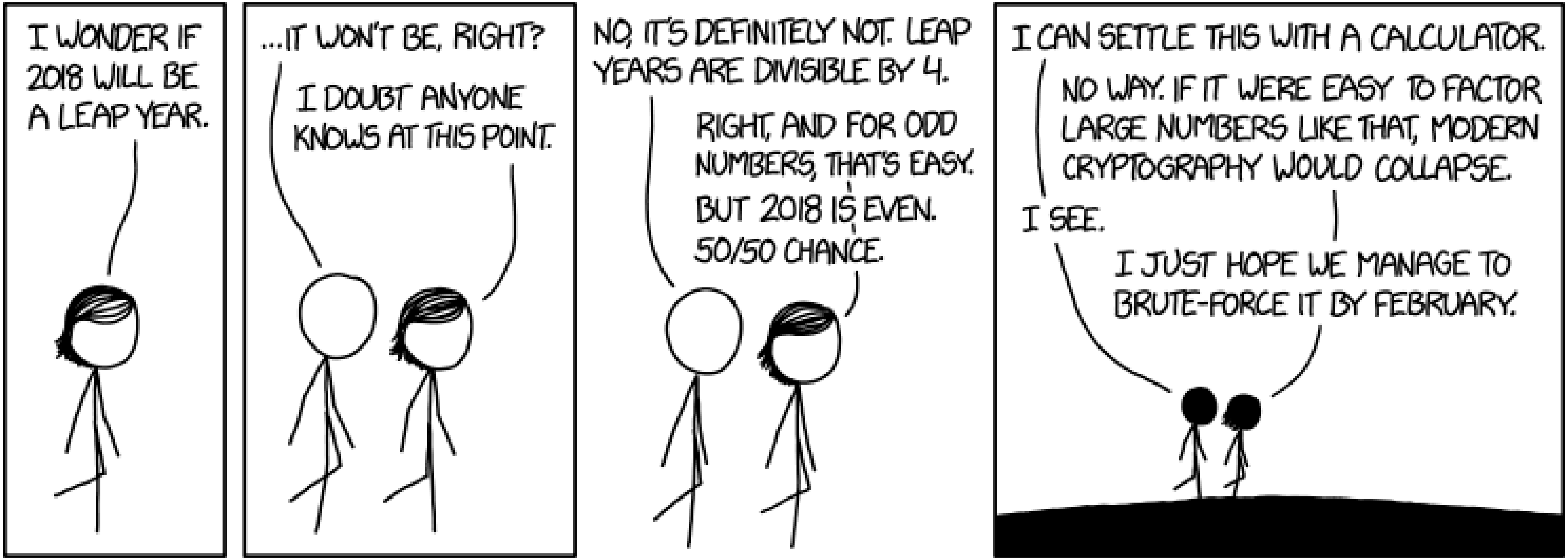
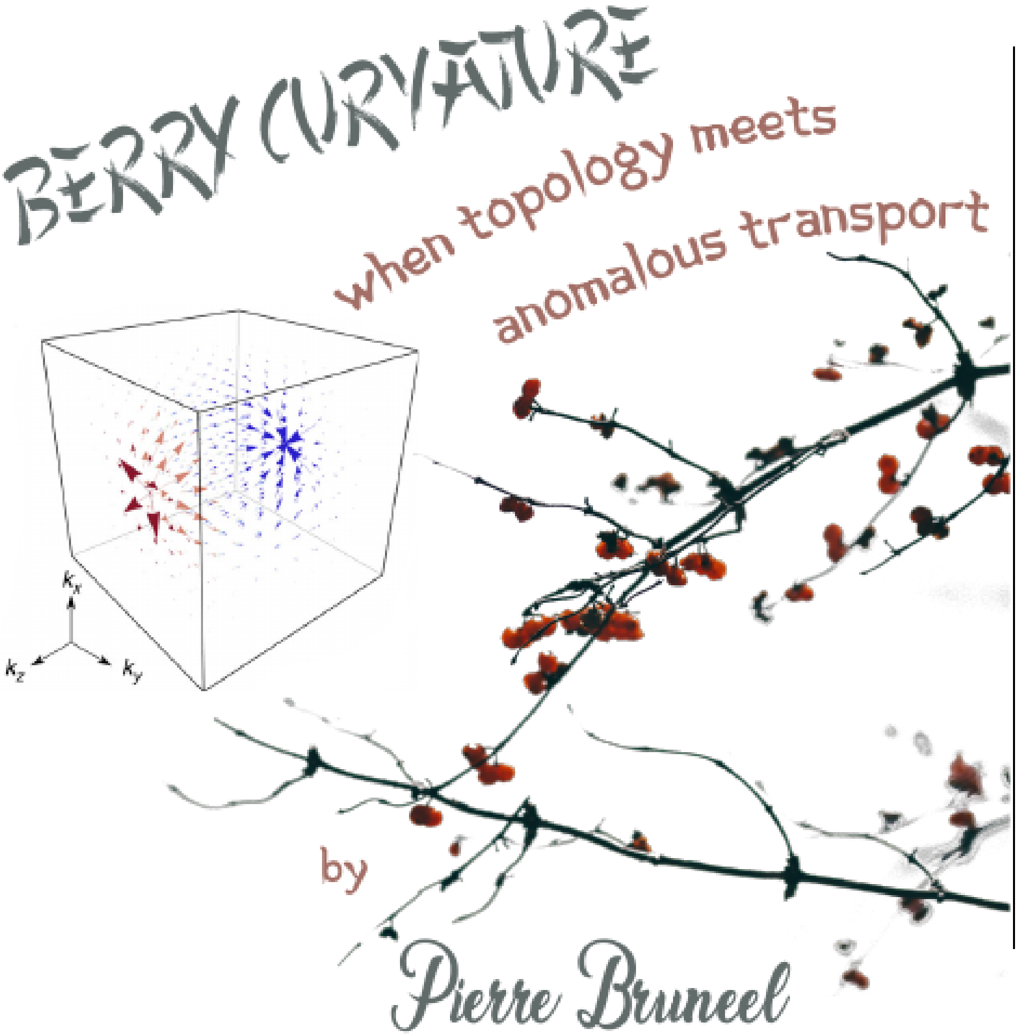
Cookies Club - February 28th



Cookies Club - February 28th, 2019



Outline

I / Introduction to band structure and electronic transport

II / Experimental result : Quantum Spin Hall Effect

III / Anomalous transport and Berry curvature

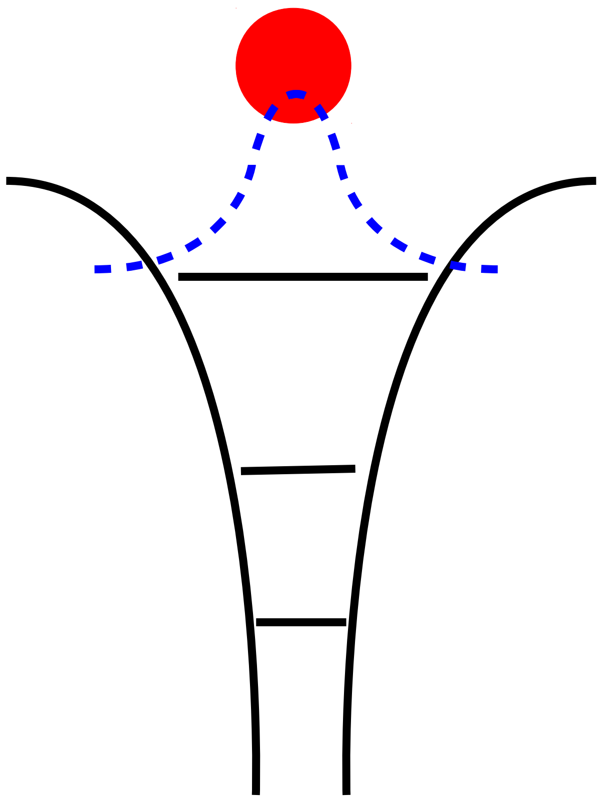
IV / A glimpse of topology

I / Schrodinger equations : from atoms to solids

Schrodinger equation describes evolution of the quantum state of a particle

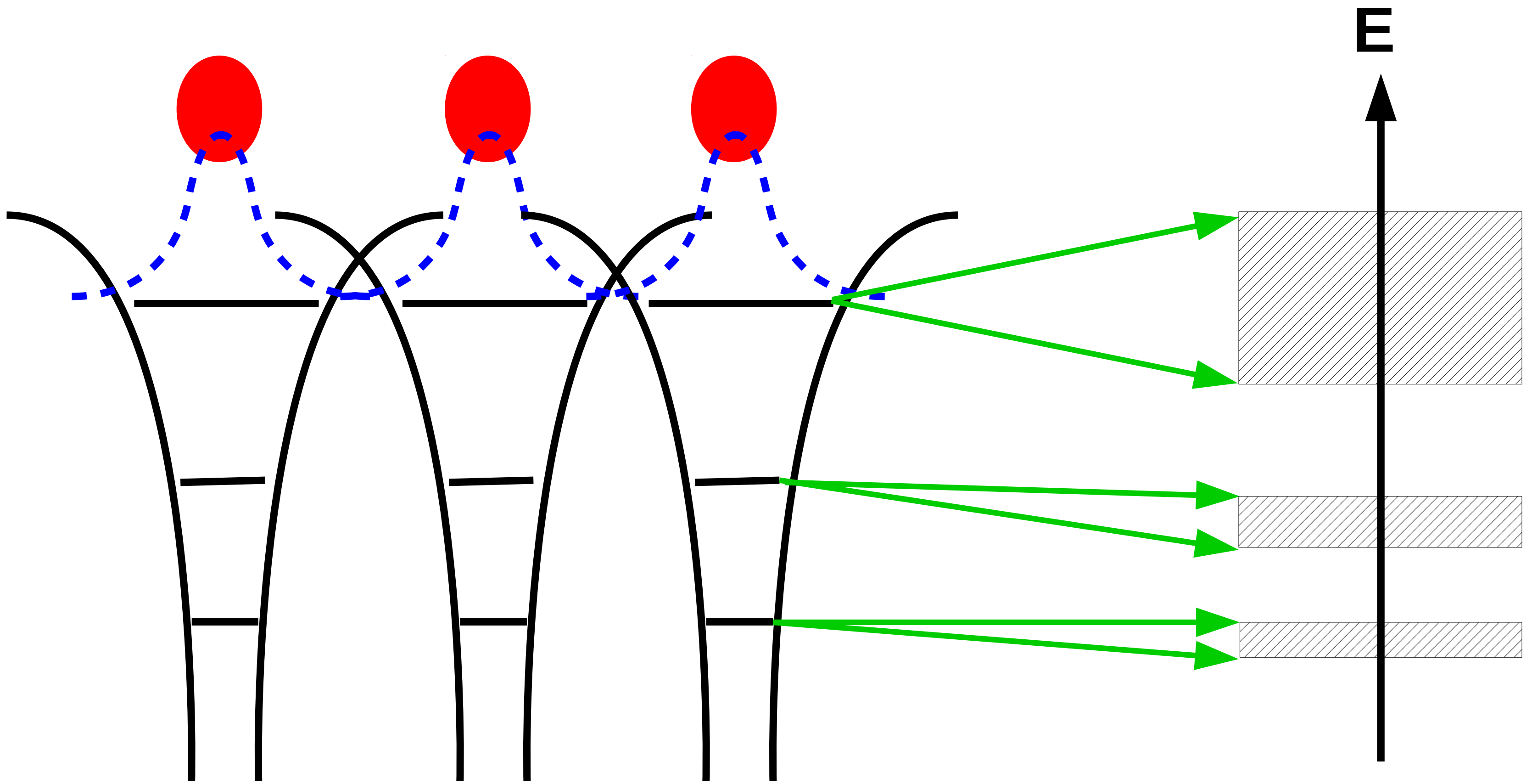
Electron trapped in a potential landscape

Discrete energy levels inside the well : index n



I / Notions of bands in solids

Hybridization of the electronic wavefunctions



I / Band structure and reciprocal space

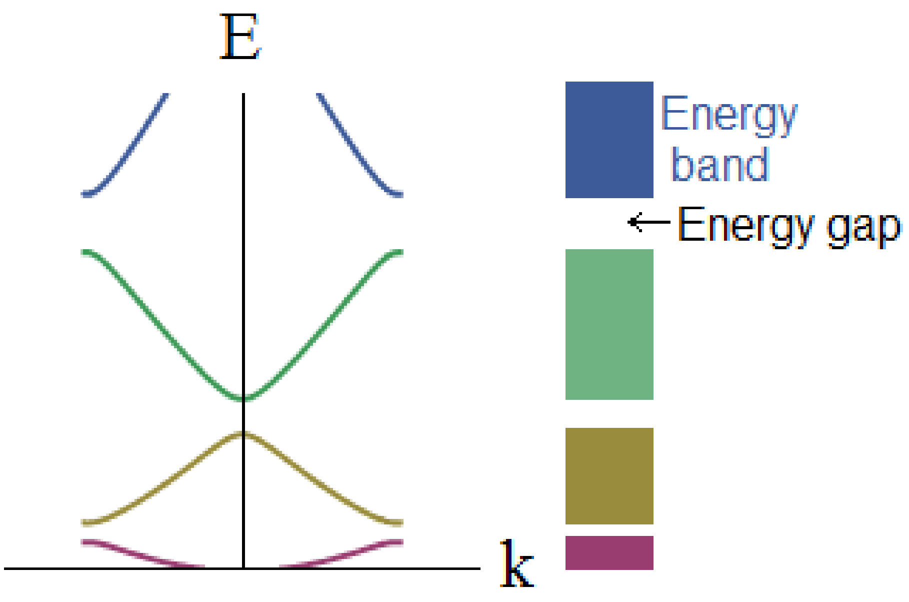
Crystal : periodic arrangement

$$\psi_{\underline{n},k}(r) = e^{ik\cdot r} u_{\underline{n}}(r)$$

$$\mathcal{H}(q) = e^{-iqr}\mathcal{H}e^{iqr}$$

Fourier transform in space

Hamiltonian for k space : band structure

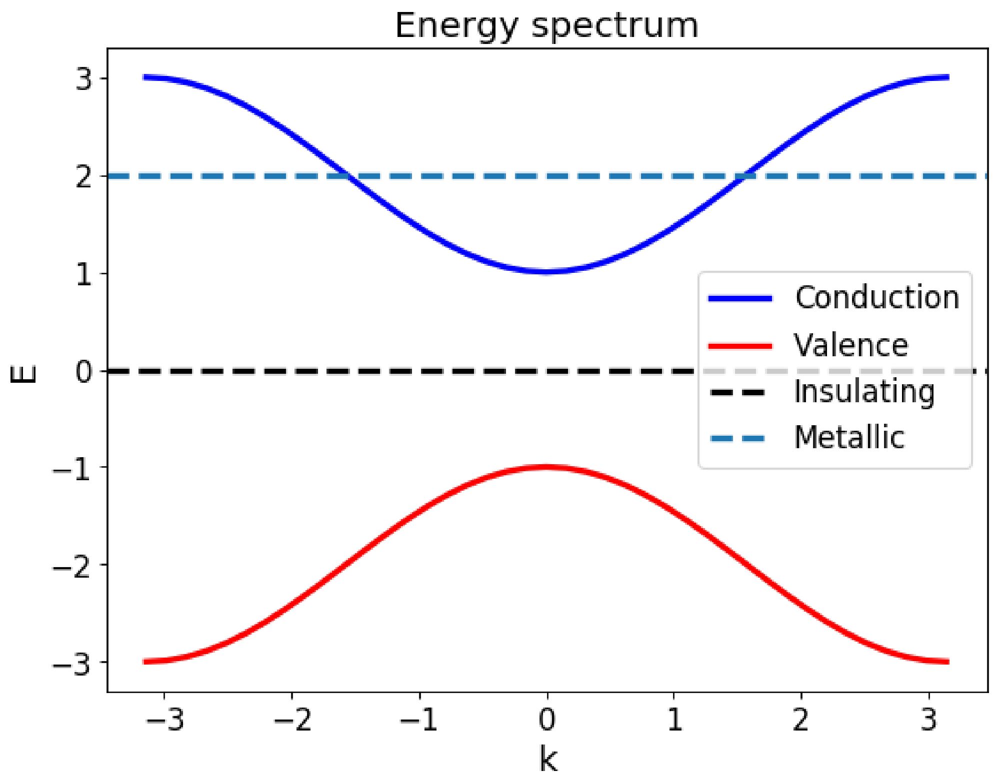


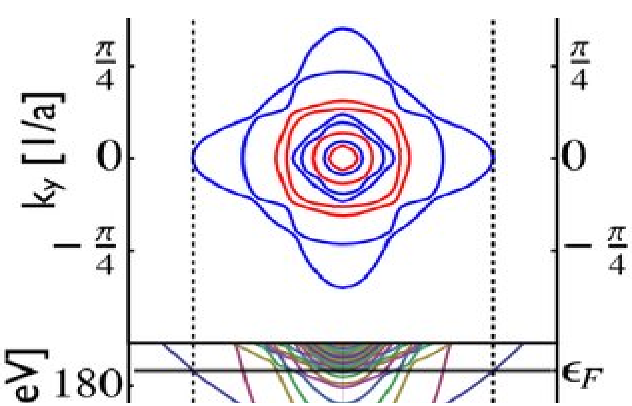
I / Metals and insulators

Electronic conductance is a property of the Fermi surface

Metal or insulator depending on the position of the Fermi energy

No transverse conductance if no magnetic field

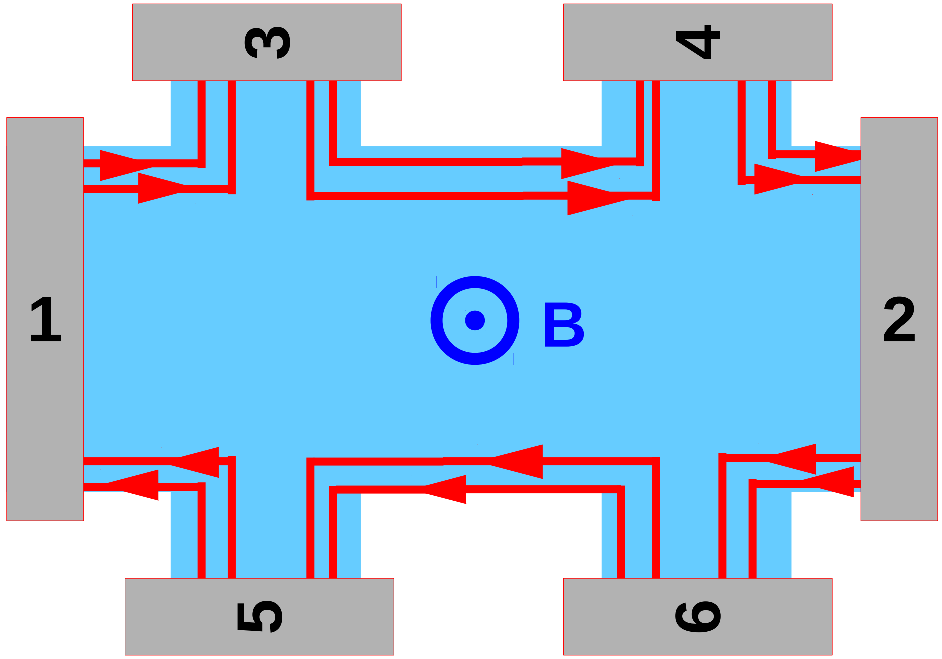




I / Hall Effect

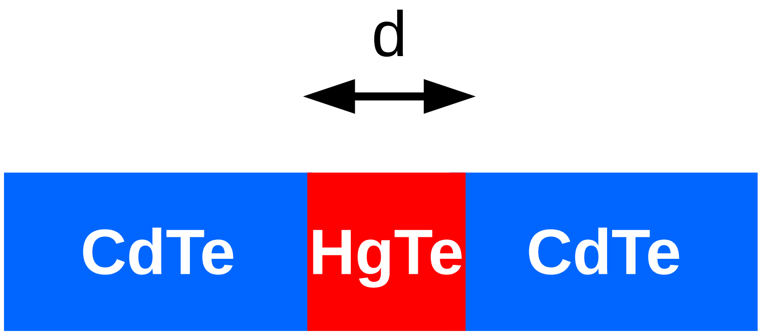
In the presence of a magnetic field : Hall Effect, it is possible to have a transverse conductance due to edge channels

Quantification of conductance because edge channels immune to backscattering

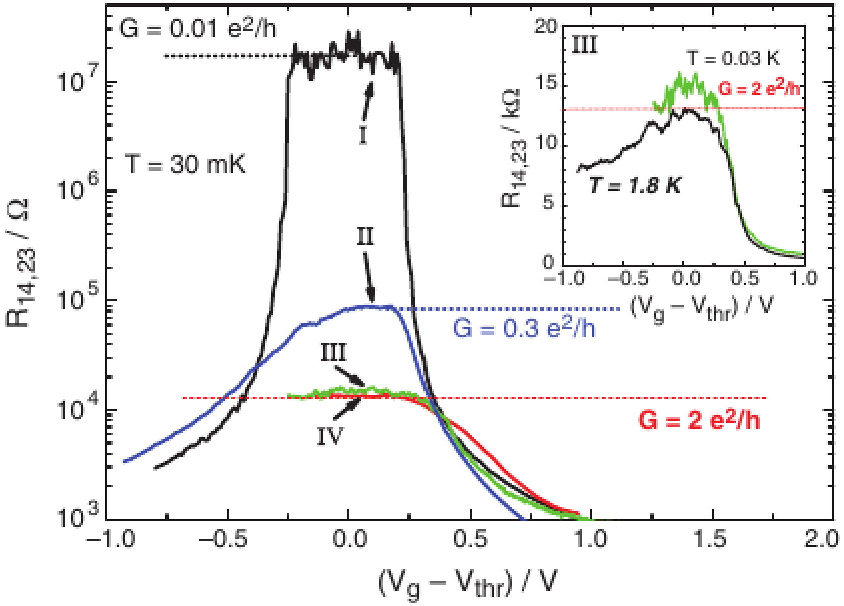


II / Quantum Spin Hall Effect

Experimental observation on sandwiches of CdTe and HgTe

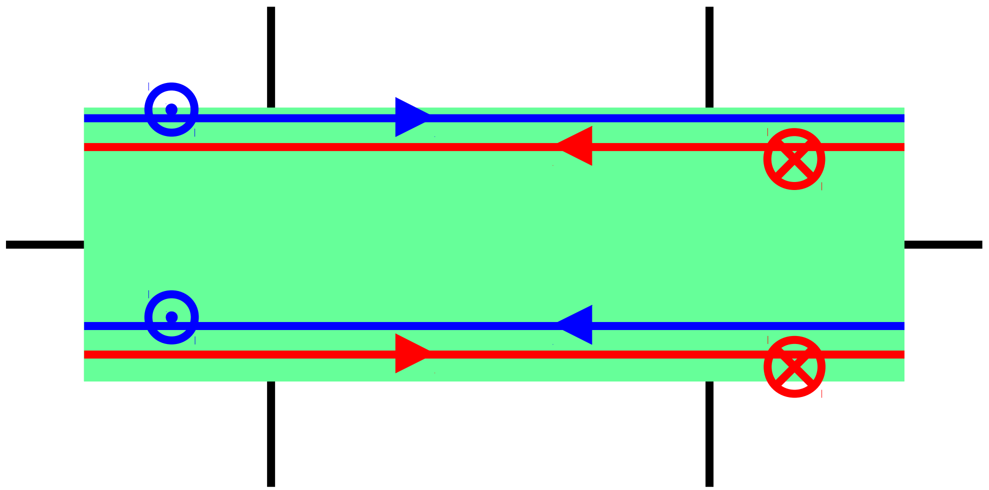


Changing the thickness of HgTe, transition from normal regime to anormal regime : apparition of quantized conductance



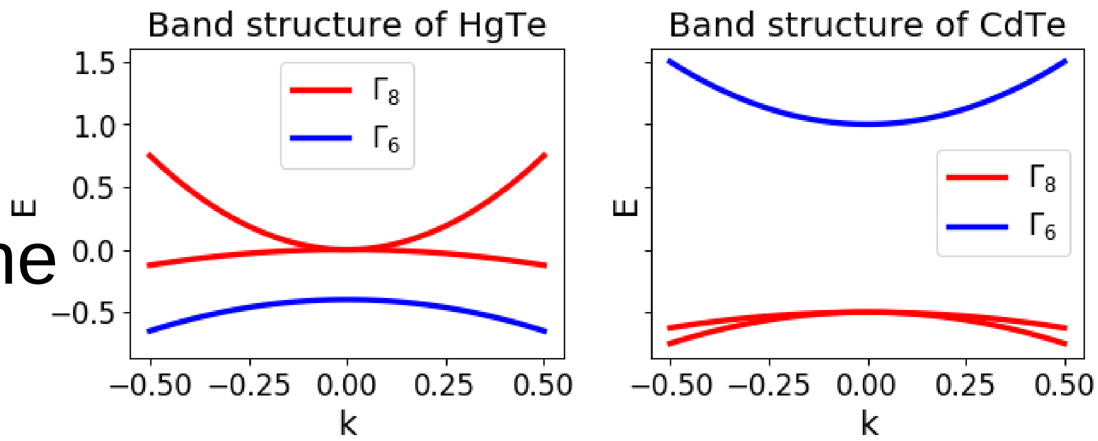
II / Quantum Spin Hall Effect

Two modes per edge with opposite spin and velocities



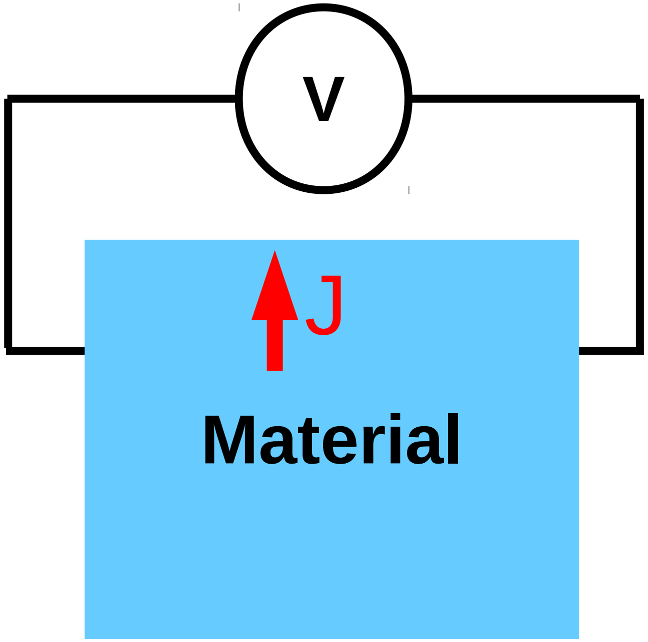
Transverse spin conductance without charge : spintronics

Inverted band structure of the two materials



III / Electric conductance

No magnetic field : no transverse conductance



Quantum correction : ***treat the velocity as an operator***

$$v_n(k) = \nabla_k \epsilon_n(k)$$

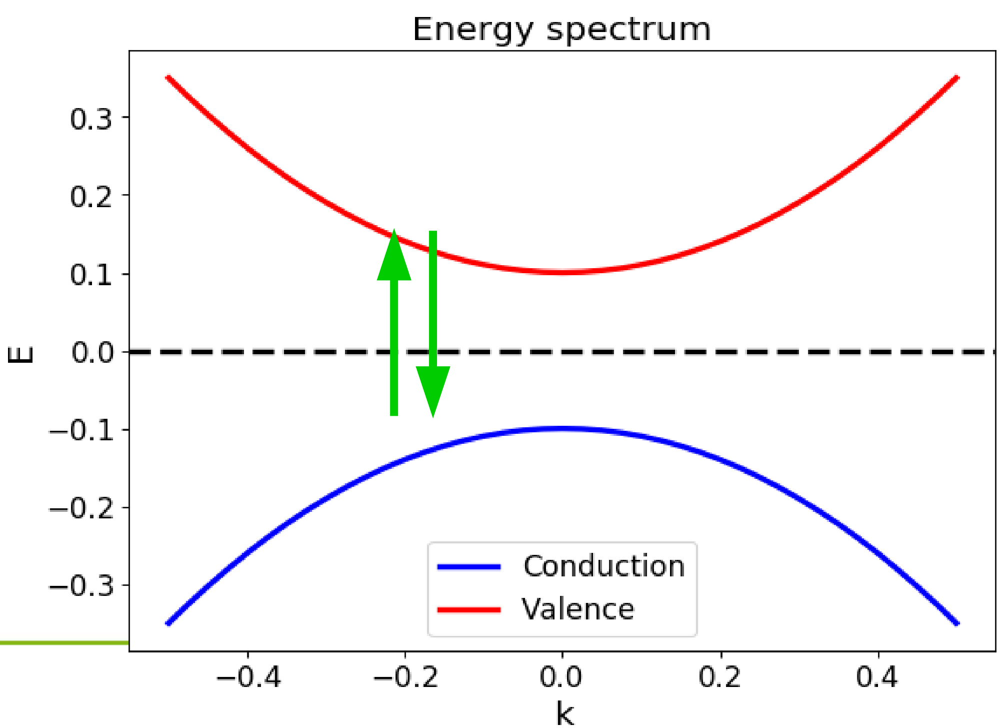
$$\hat{v}(k) = \nabla_k H(k)$$

III / Quantum corrections to electric current

$$\delta|n_k\rangle = \sum_{n' \neq n} \frac{\langle n'_k|eEx|n_k\rangle}{(\epsilon_{nk} - \epsilon_{n'k})} |n'_k\rangle$$

Allow for ***interband transition*** processes

Response of the system is a mean value of the current



III / Berry curvature and anomalous conductivity

$$\sigma^{xy}(\mu) = \int \frac{dk^2}{4\pi^2} \Omega_n(k) f(\epsilon_{nk} - \mu)$$

Filling

Brillouin zone

Occupied states

Effect of the whole band geometry, ***non-local***

Varies with the occupation of the band

III / Quantum correction to the conductance

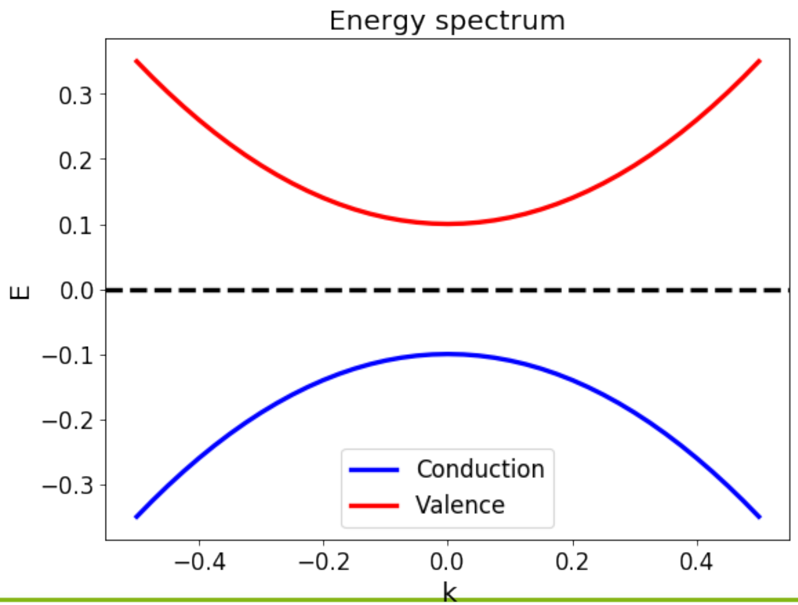
Compute the new mean value of the velocity operator and use $x \leftrightarrow i\partial_k$

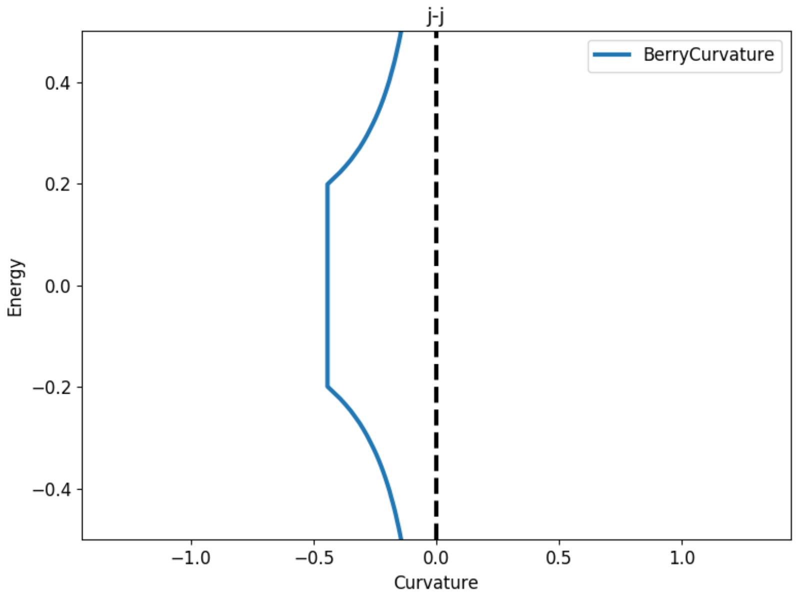
Apparition of the ***Berry curvature***

$$\Omega_n^{\mu\nu}(k) = Im\left[\sum_{n'\neq n} \frac{\langle n_k|\partial_\mu \mathcal{H}|n'_k\rangle\langle n'_k|\partial_\nu \mathcal{H}|n_k\rangle}{\underline{(\epsilon_{nk} - \epsilon_{n'k})^2}}\right]$$

III / A simple example : a two level system

$$\mathcal{H} = \begin{bmatrix} \Delta & k_x - ik_y \\ k_x + ik_y & -\Delta \end{bmatrix}$$





III / Berry curvature and symmetries

Time reversal symmetry $\Omega_n(-k) = -\Omega_n(k)$

Inversion symmetry $\Omega_n(-k) = \Omega_n(k)$

Total Berry curvature $\sum_n \Omega_n(k) = 0$

IV / Analogy with magnetism

Magnetism

Real space : r

$$\nabla_r \times A(r) = B(r)$$

$$\vec{F} = -e\vec{v} \times \vec{B}$$

Berry

Reciprocal space : k

$$\nabla_k \times \mathcal{A}_n(k) = \Omega_n(k)$$

$$\vec{v}^{a}_{nk} = \frac{e}{\hbar}\vec{E} \times \vec{\Omega}_n(k)$$

IV / Chern number and quantization

$$C_n = \int_{BZ} \frac{d^2k}{2\pi} \Omega_n(k) \in \mathbb{Z}$$

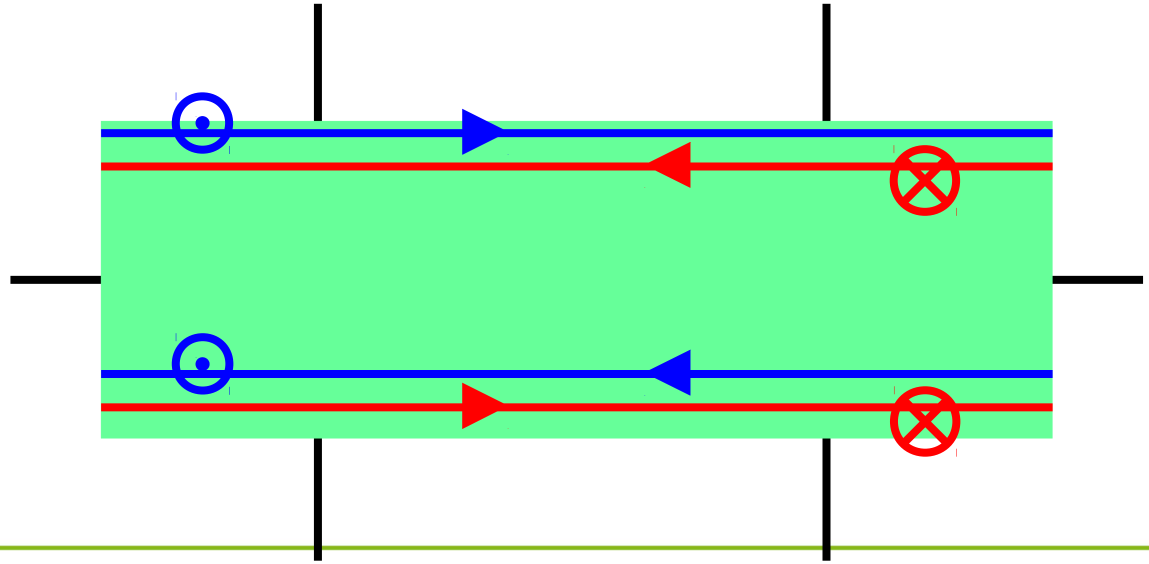
It is only linked to the topology of the space spanned by the eigenvectors

Protected by the gap of the system

IV / Chern number and quantization

Chern number : number of edge states that will flow. Sign indicates direction

Case of QSHE : 2 copies of previous



Take-home message

Conductivity in metals : band structure and shape of its Fermi surface

Quantum corrections due to interband transitions can lead to anomalous transport : Berry curvature

Anomalous transport comes from topological invariants of the band structure : robustness of quantization

References

Bernevig, B. A., Hughes, T. L., & Zhang, S. C. (2006). Quantum spin Hall effect and topological phase transition in HgTe quantum wells. Science, 314(5806), 1757-1761.

König, M., Wiedmann, S., Brüne, C., Roth, A., Buhmann, H., Molenkamp, L. W., ... & Zhang, S. C. (2007). Quantum spin Hall insulator state in HgTe quantum wells. Science, 318(5851), 766-770.

Xiao, D., Chang, M. C., & Niu, Q. (2010). Berry phase effects on electronic properties. Reviews of modern physics, 82(3), 1959.

IV / Link with topology and differential geometry

$$\Omega_n(k) = i\langle \nabla_k n_k| \times |\nabla_k n_k\rangle$$

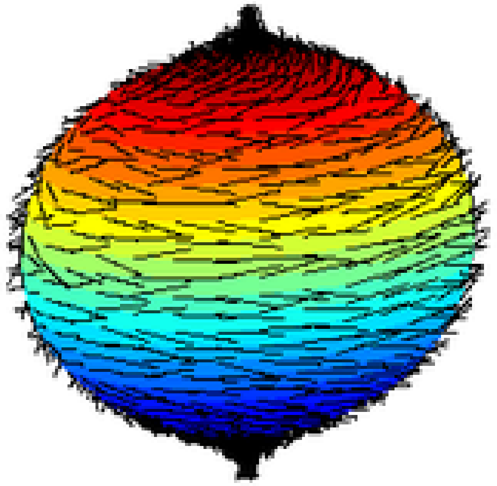
Berry curvature can be interpreted as an object from differential geometry

Appears when differentiating quantities along with the Berry connection

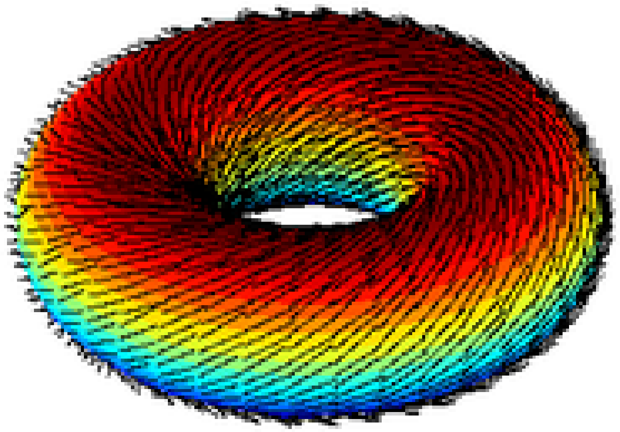
$$A_n(k) = i\langle n_k|\nabla_k|n_k\rangle$$

IV / Topological obstruction

Hairy ball theorem : it is impossible to put hair on a whole sphere without having a singular point



However it is possible on a donut for instance



Topological nature of the object is different : genus of the surface

IV / Chern number and quantization

$$C_n = \int_{BZ} \frac{d^2k}{2\pi} \Omega_n(k) \in \mathbb{Z}$$

It is only linked to the topology of the space spanned by the eigenvectors

For a 2-level system, the eigenvectors live on a sphere (Bloch sphere)

It is equal to the number of times the map wraps the sphere

QSHE

$$\mathcal{H} = \begin{bmatrix} \Delta & k_x - ik_y & 0 & 0 \\ k_x + ik_y & -\Delta & 0 & 0 \\ 0 & 0 & \Delta & -k_x - ik_y \\ 0 & 0 & -k_x + ik_y & -\Delta \end{bmatrix}$$

Berry curvature and semi classical approach ? Projection ? SO ? Spin ?

Notion of projection, projected physics on one band

How to project out higher states

The position operator is ill defined in Bloch Hamiltonian

Berry curvature

Berry curvature