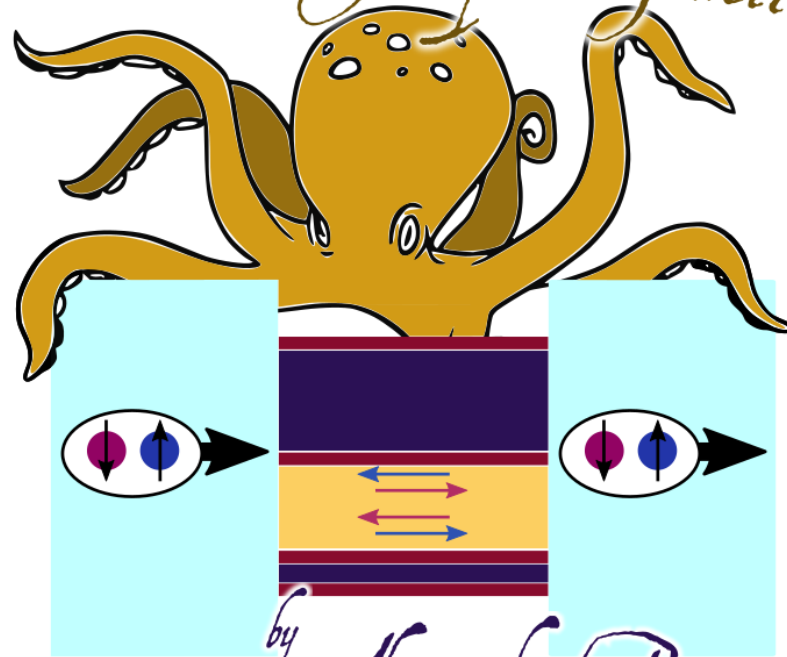
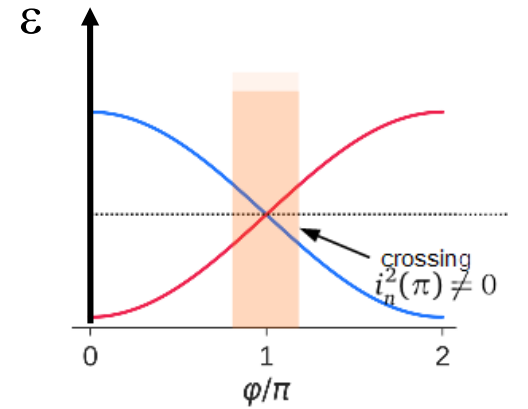
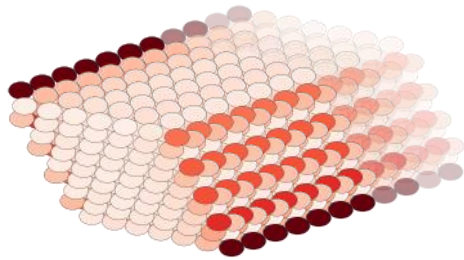
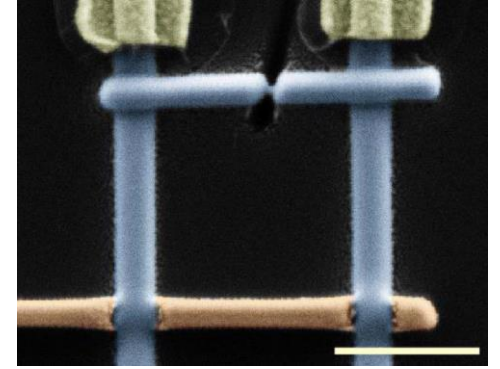


*Quantum interferences
in Bismuth nanowires
Josephson junctions*

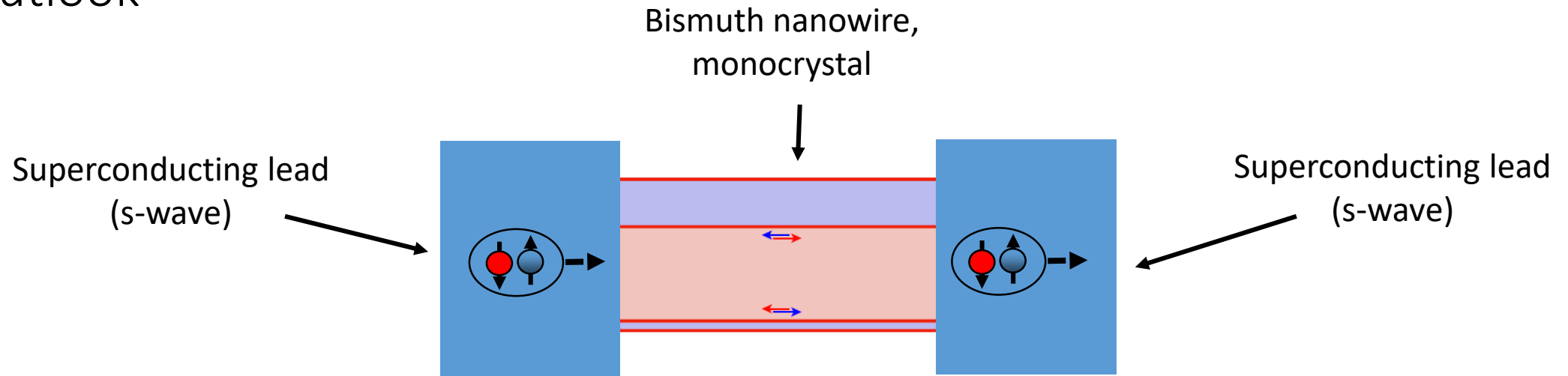


by Alexandre Bernard

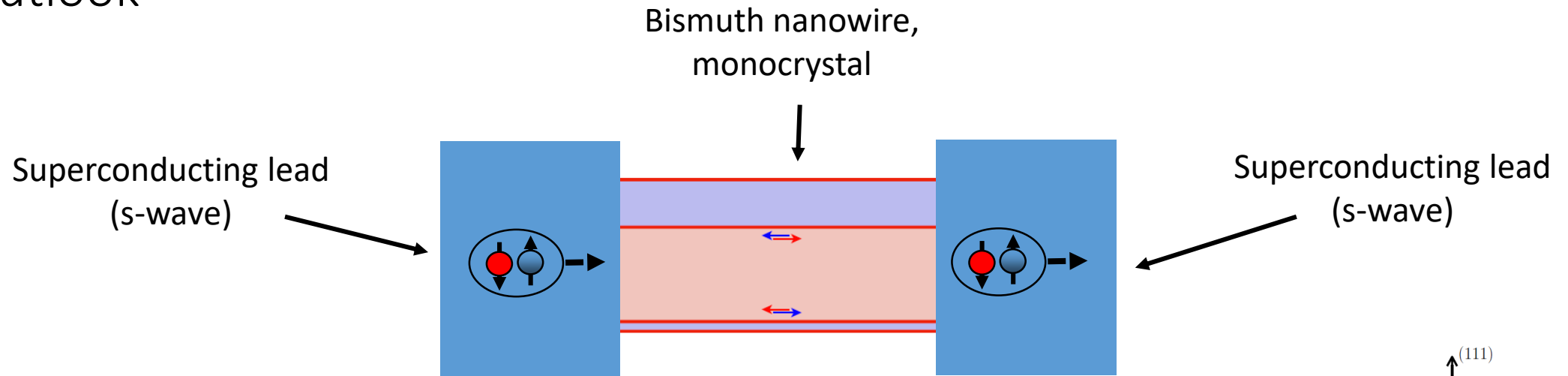


and
Sophie Guéron, Anil Murani, Alik Kasumov, H el ene Bouchiat
MESO group, Laboratoire de Physique des Solides, Orsay, France

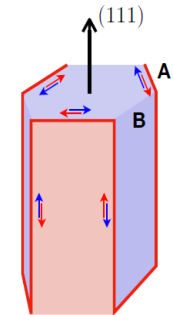
Outlook



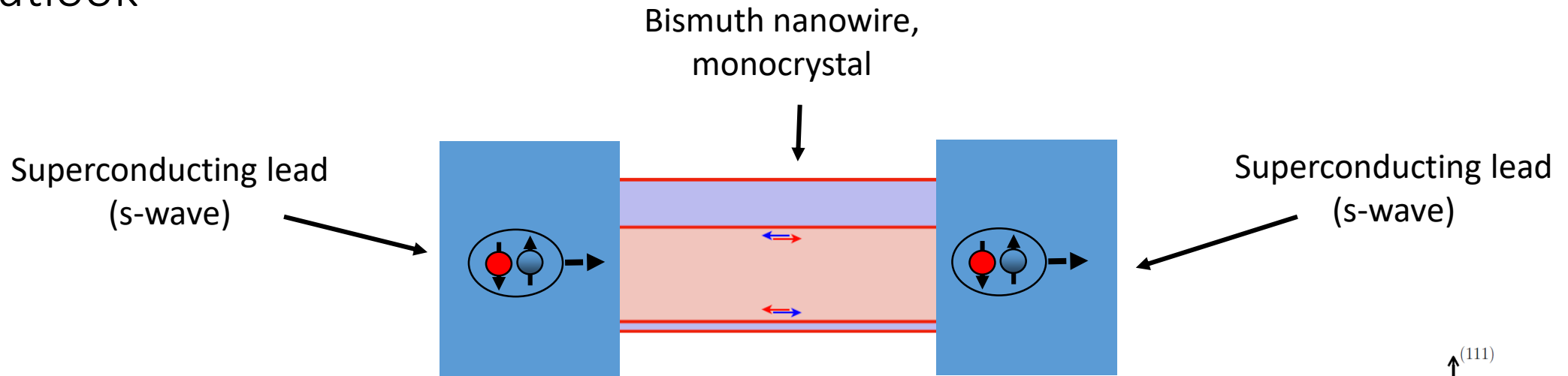
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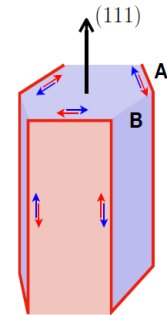
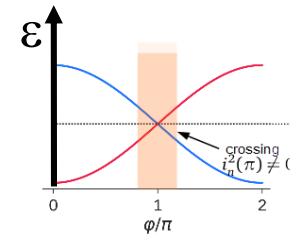
- Topological helical edge states in Bismuth, in a nutshell



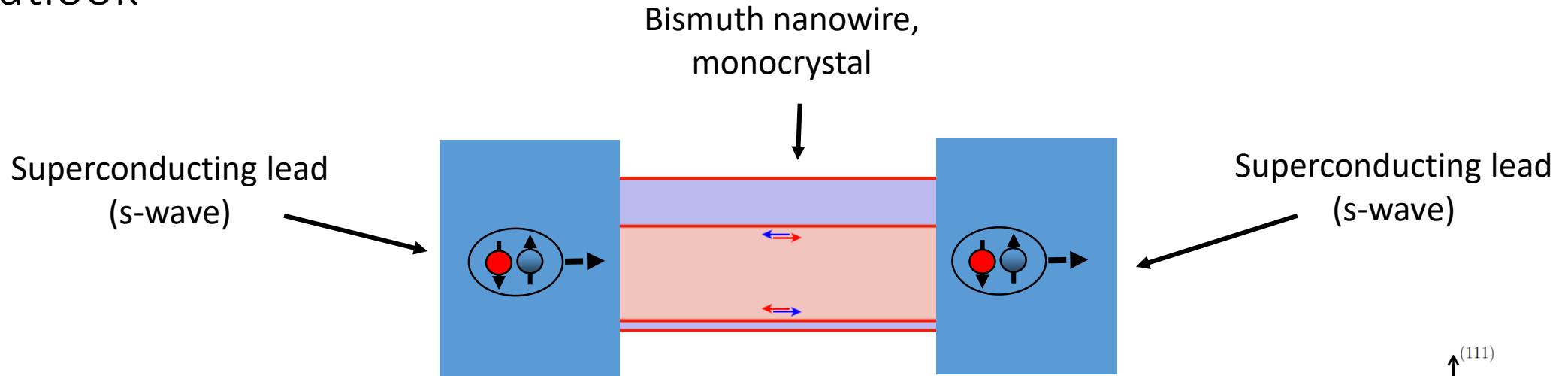
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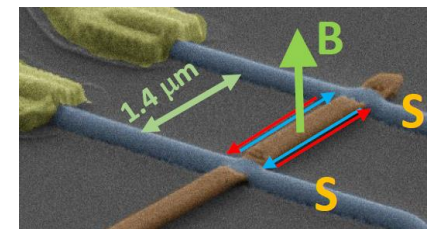
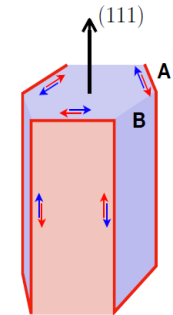
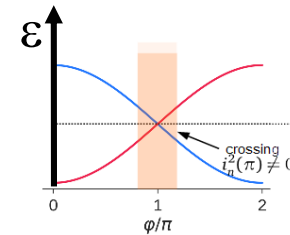
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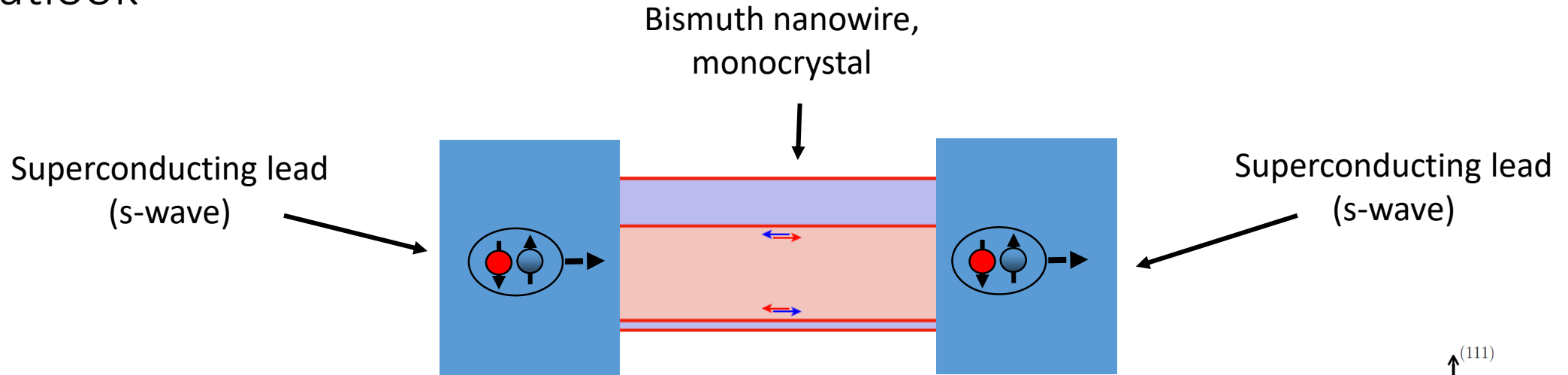
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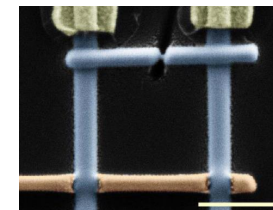
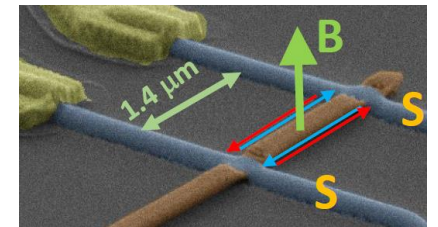
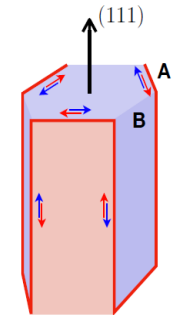
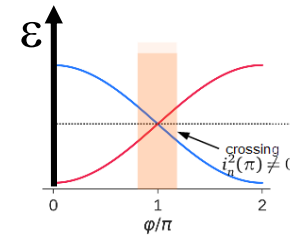
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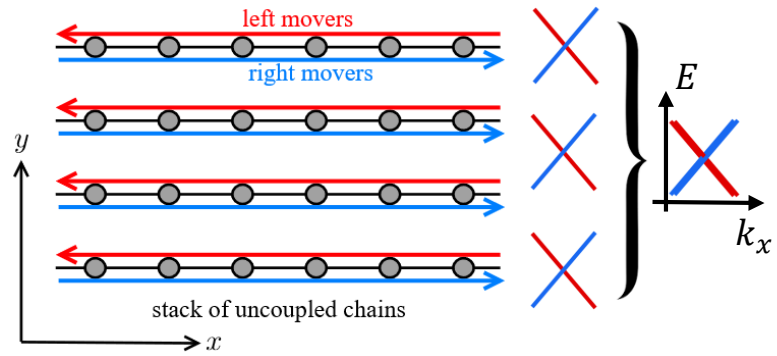
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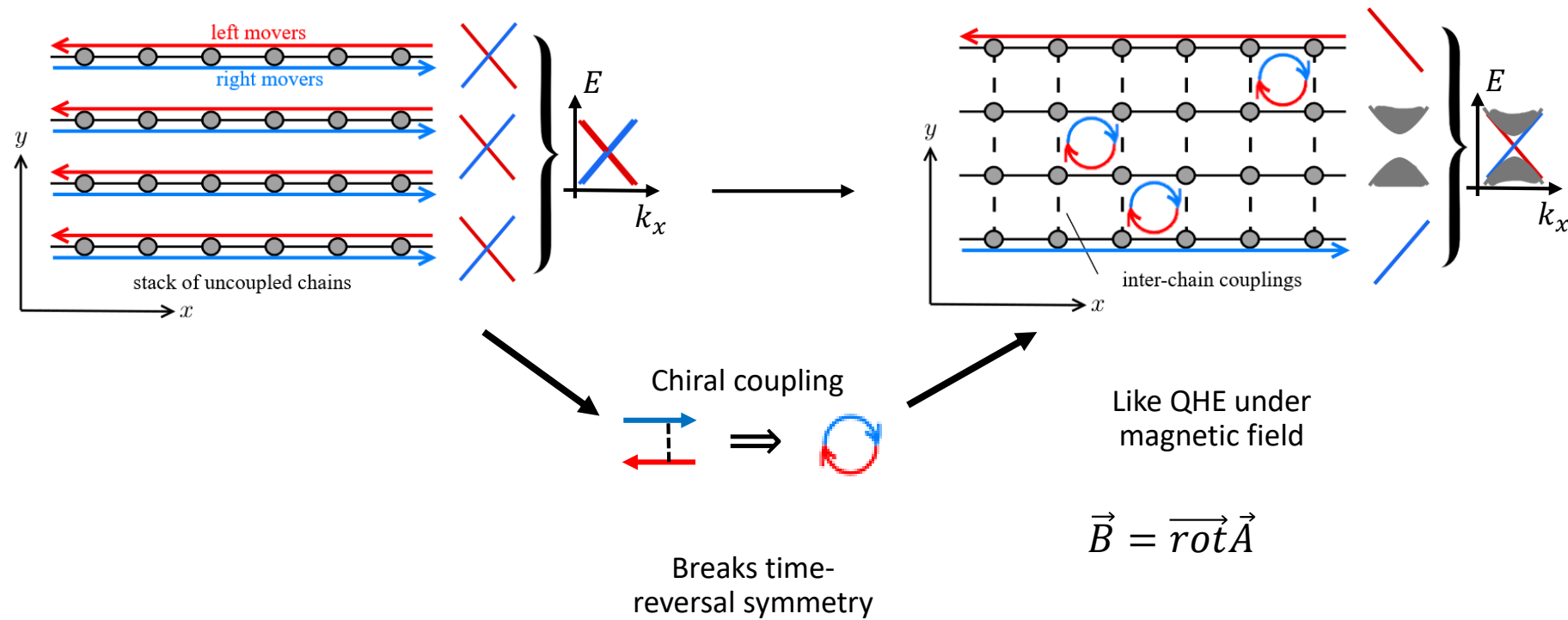
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- Supercurrent vs phase, another hint for topologically protected states



Topological insulator with chiral edge state



Topological insulator with chiral edge state

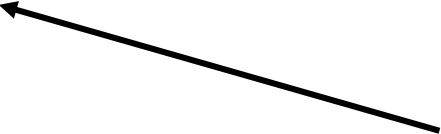


Topological insulator without magnetic field

$$H_{SO} = i \sum_{\langle\langle i,j \rangle\rangle, s, s'} \lambda_{SO}^i \cdot v_{ij} \cdot [\hat{S}_z]_{ss'} \cdot \hat{c}_{is}^\dagger \cdot \hat{c}_{js'}$$

Spin-Orbit interaction, a purely relativistic effect

2nd order terms,
two 1st neighbors hops,
effectively 2nd neighbors hops



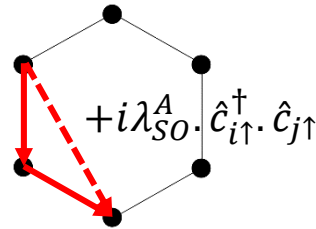
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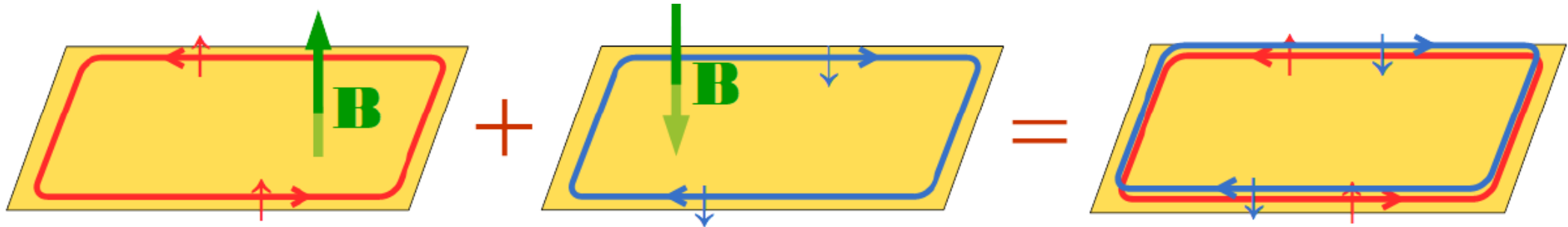
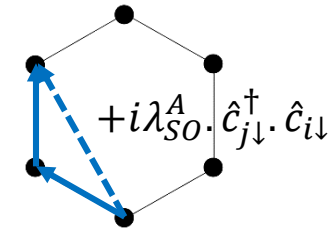
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Spin up-up : $[\hat{S}_z]_{\uparrow\uparrow} = +1$
Positive rot : $v_{ij} = +1$

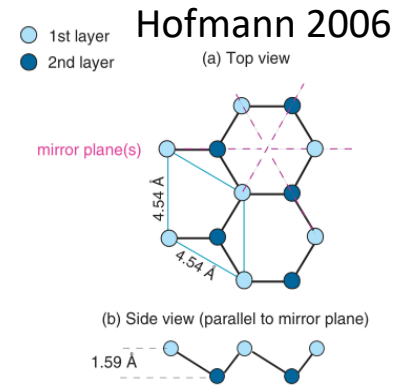


Spin down-down : $[\hat{S}_z]_{\downarrow\downarrow} = -1$
Negative rot : $v_{ji} = -1$

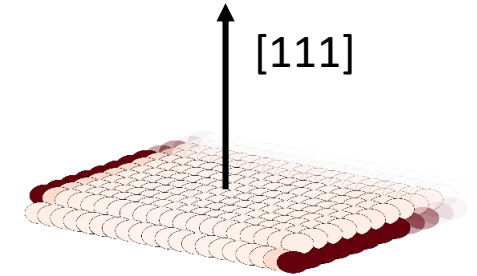
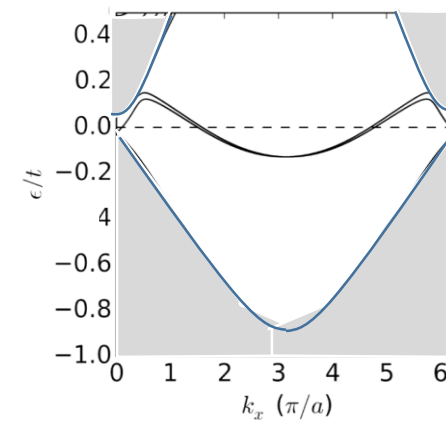


Topologically protected states in ordered Bismuth nanowires ?

- High SOI, key ingredient for TIs with TRS
- Bi 111 bilayer ribbon is a 2D TI with helical edge states (Murakami, PRL 2006)

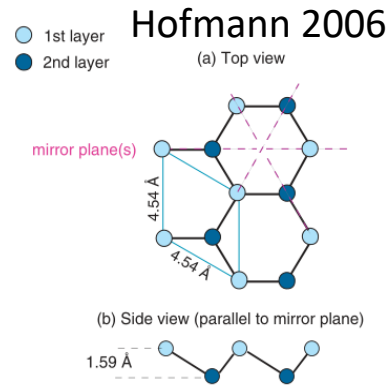


(111) Bi bilayer

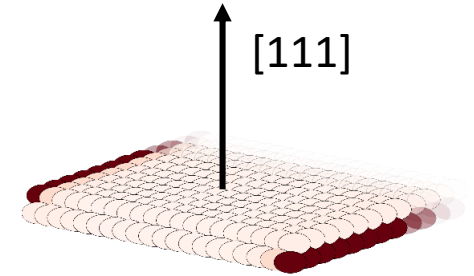
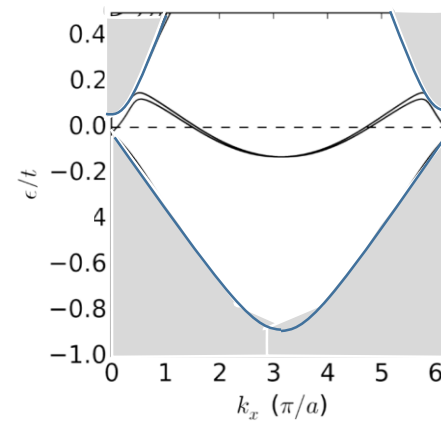


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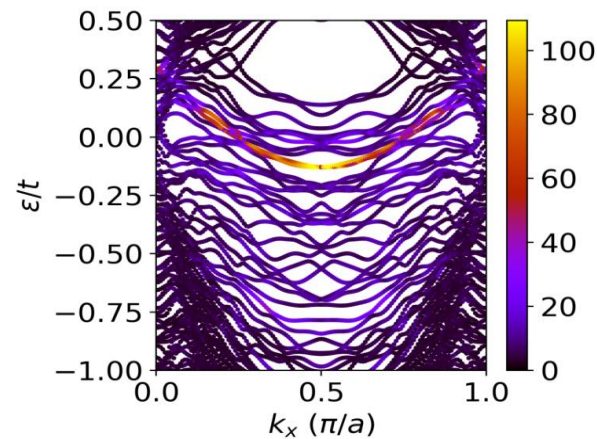


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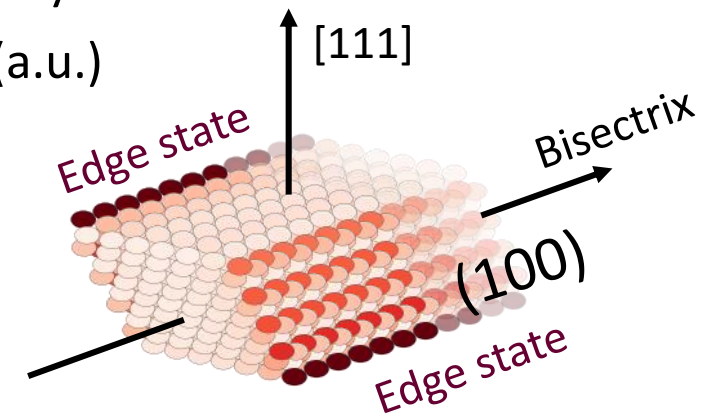
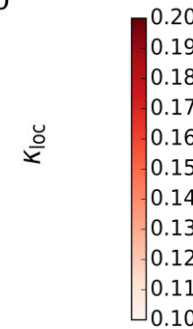


- 5 Bi 111 bilayers is somewhat a 2D TI, surface states + 1D hinge states (Murani et al., Nat. Comm. 2017)

(111) Bi 5 bilayers



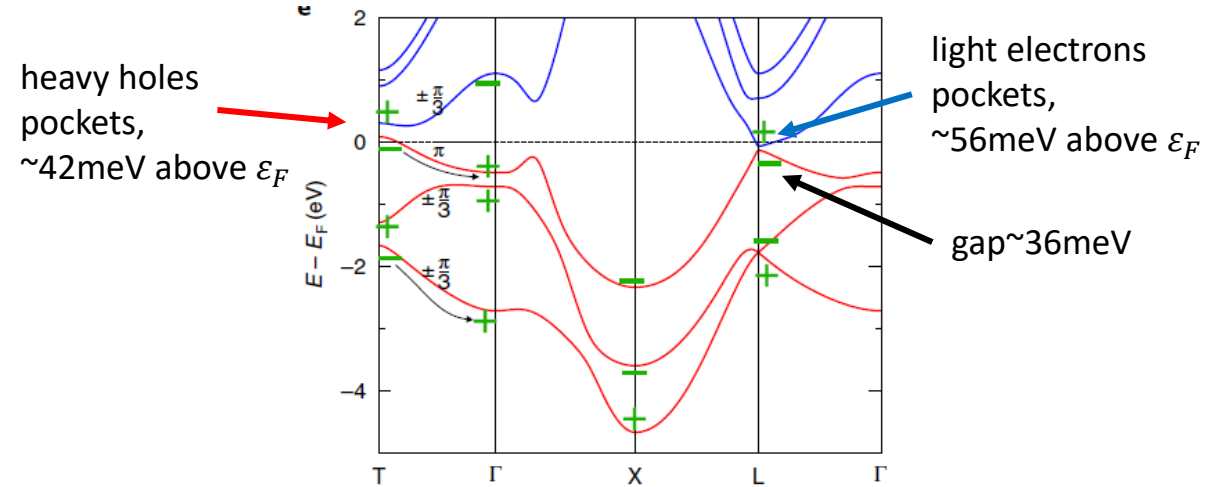
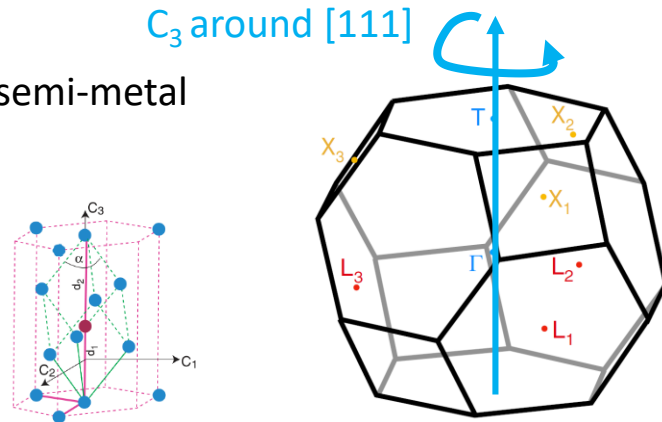
LDOS (a.u.)



- >8 Bi 111 bilayers predicted not TI anymore (according to Liu et al., PRL 2011)

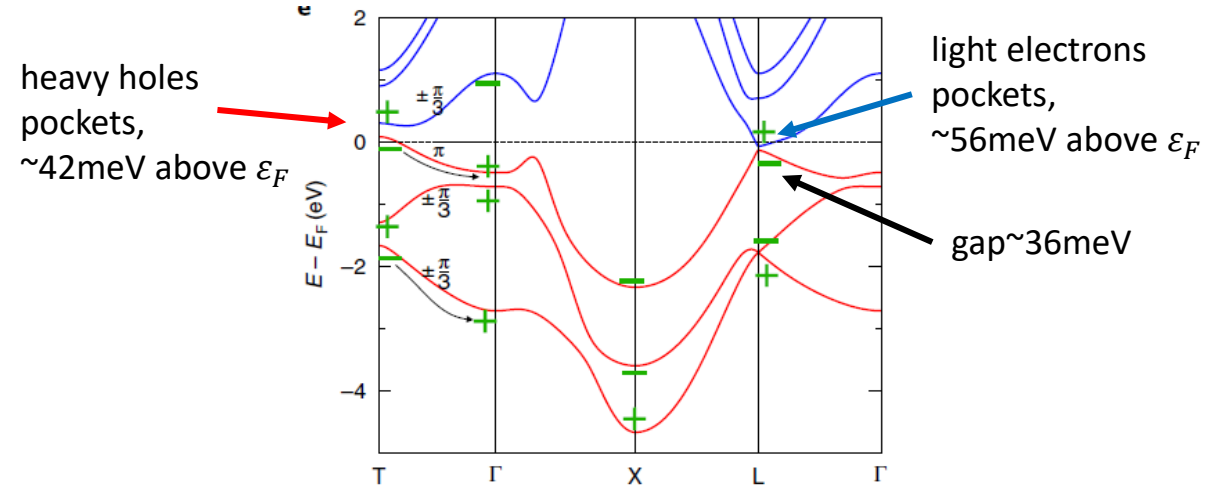
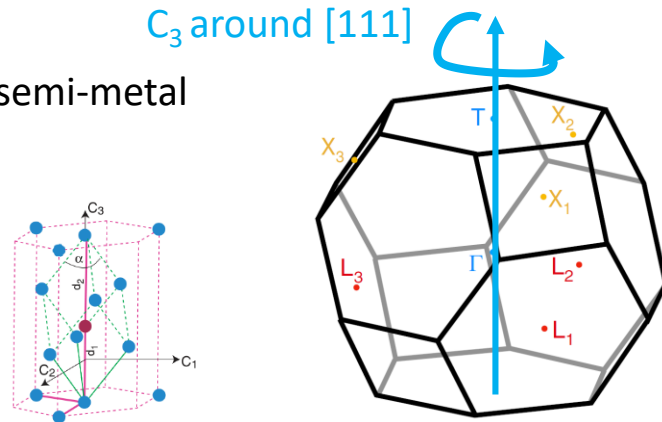
Topologically protected states in ordered Bismuth nanowires ?

- Bulk Bi not 3D TI, semi-metal



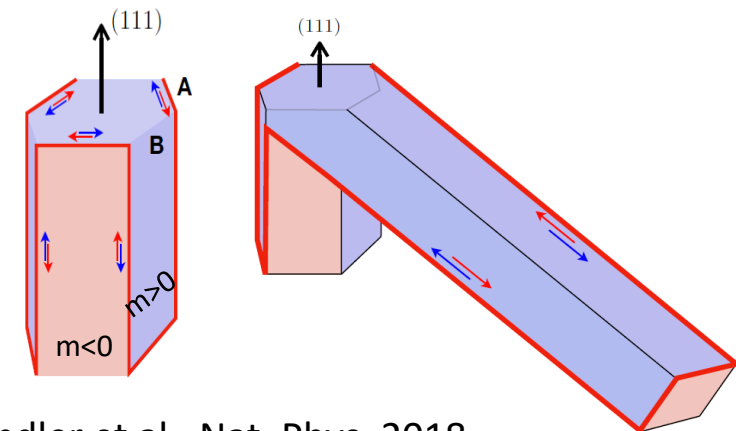
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- But... C_3 and I symmetries with topologically non-trivial subspaces, allow for higher (second) order topology

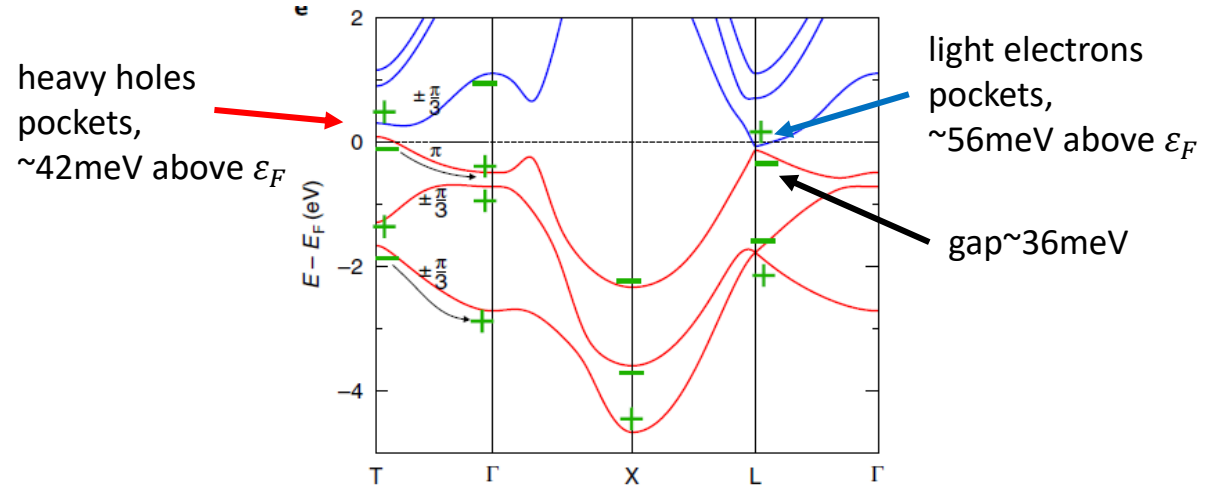
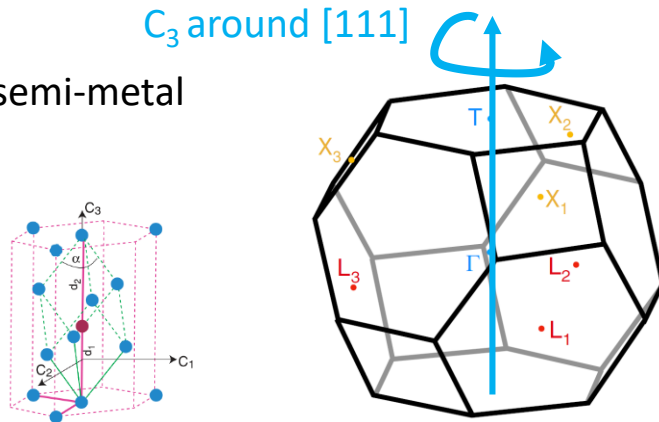
=> pure monocrystalline Bi is predicted to be a HOTI
 => no gapless surface states, but helical hinge states



Schindler et al., Nat. Phys. 2018

Topologically protected states in ordered Bismuth nanowires ?

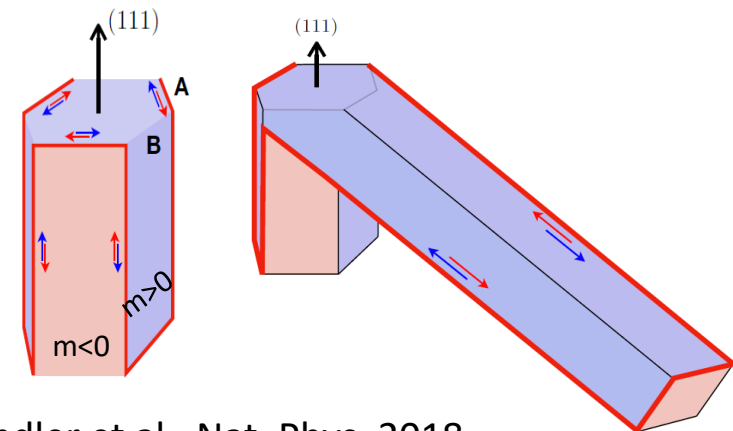
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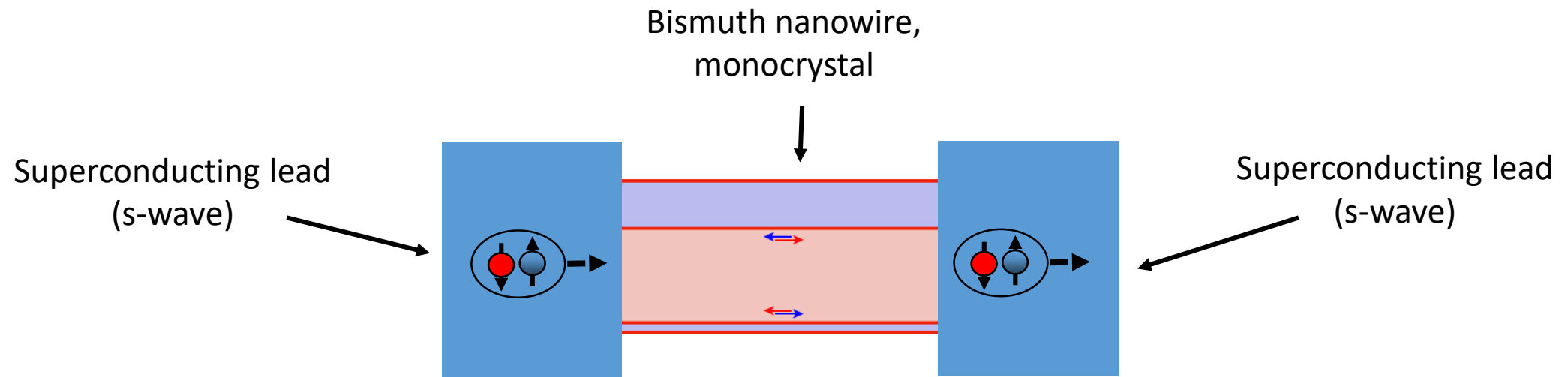
=> pure monocrystalline Bi is predicted to be a HOTI
 => no gapless surface states, but helical hinge states

- But... Bi is still a semi-metal and not an insulator
- The geometry of the samples is difficult to control, and electron transport may be affected by various finite size effects
- The HOTI picture is recent, and it is not clear what to expect experimentally

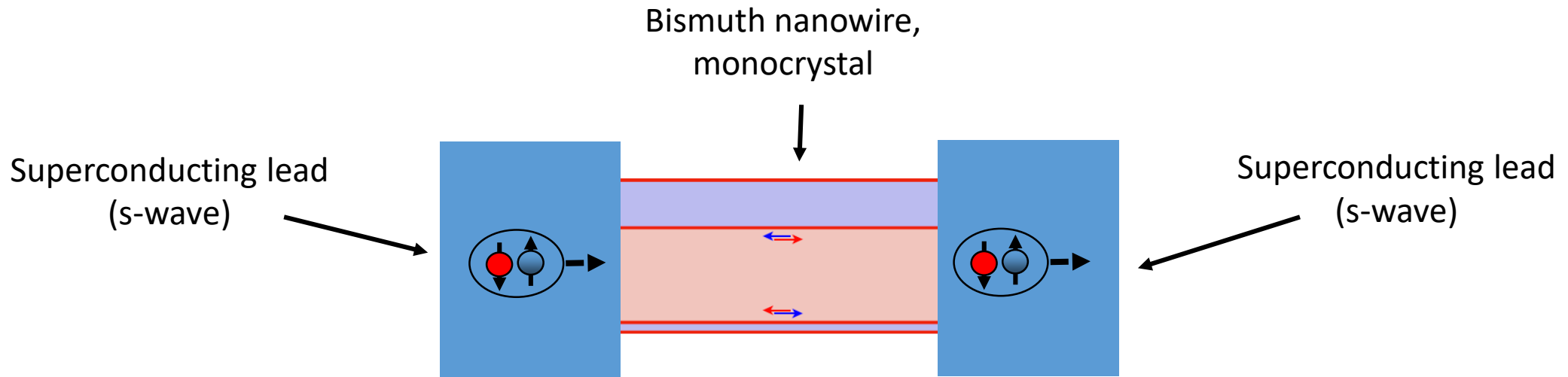


Schindler et al., Nat. Phys. 2018

Why using superconducting proximity effect ?



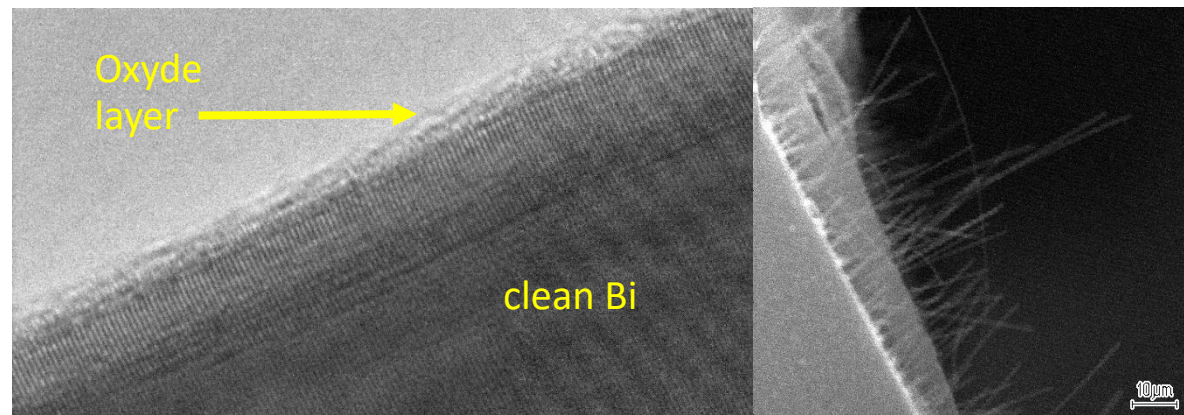
Why using superconducting proximity effect ?



Allow to:

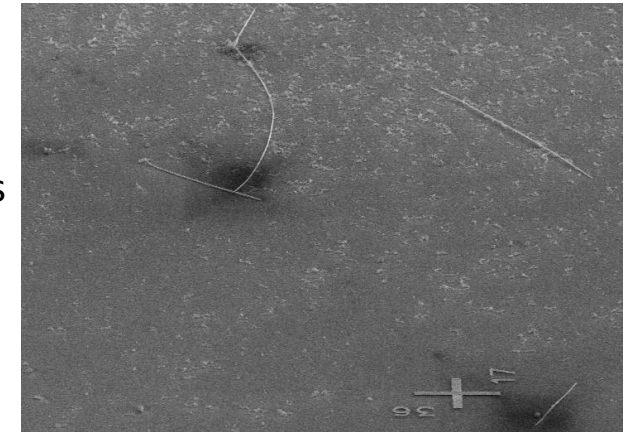
- Reduce the contribution of diffusive states compared to the ballistic ones (phase coherence of the e-h pair required)
- Spatial distribution of the supercurrent through the junction with **critical current-flux relation**
=> **is there 1D states ? Where ?**
- Dependence of the energy on the superconducting phase with **supercurrent-phase relation**
=> **are the states perfectly transmitted, with perfect crossing at $\varphi = \pi$ as expected for topologically protected states ?**

Sample fabrication



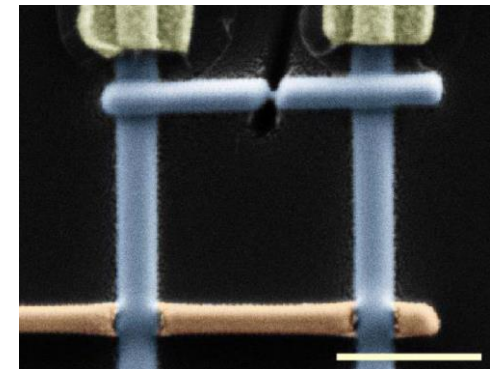
Bismuth nanowires:

- Previously **grown by sputtering on a hot SiO₂ substrate covered with a thin wetting layer** and picked up and dropped with a clean piece of wiper
=> **nice ordered nanowires of D~200nm L~10µm** (IMT RAS, Chernogolovka)
- Now **grown by sputtering on a cold substrate (reach ~70°)**, deposited with 10ns UV laser pulses
=> **nice ordered nanowires of D~100nm L~20µm, but potentially some strain**
- Checked with Transmission Electron Microscope on the edges (IMT RAS, Chernogolovka)
- Selected and checked with Electron BackScattering Diffraction (ICMMO, Orsay)

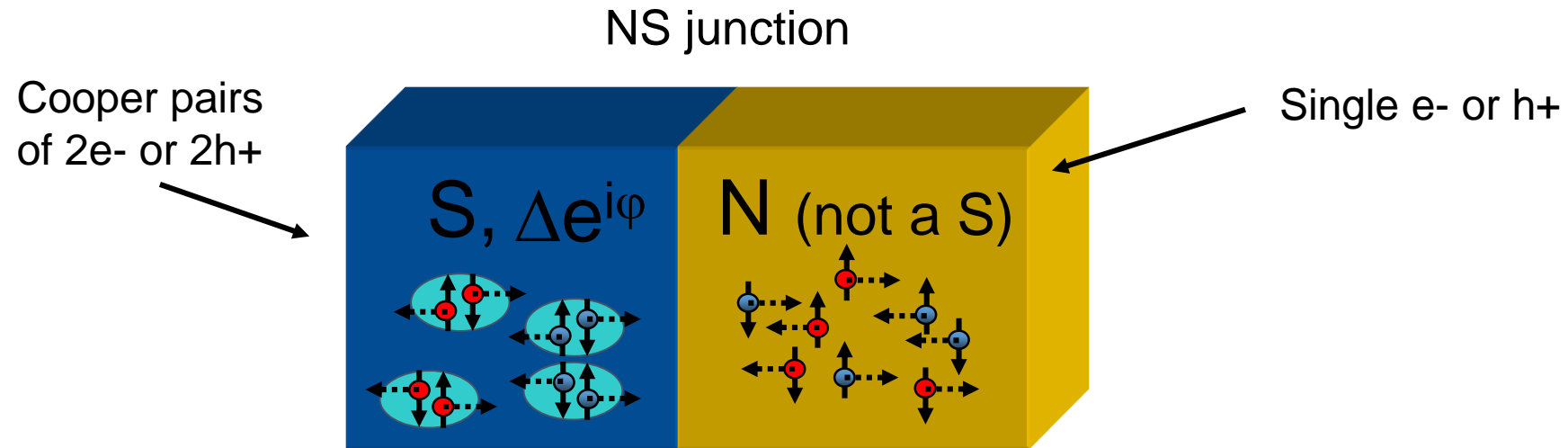


Contacts:

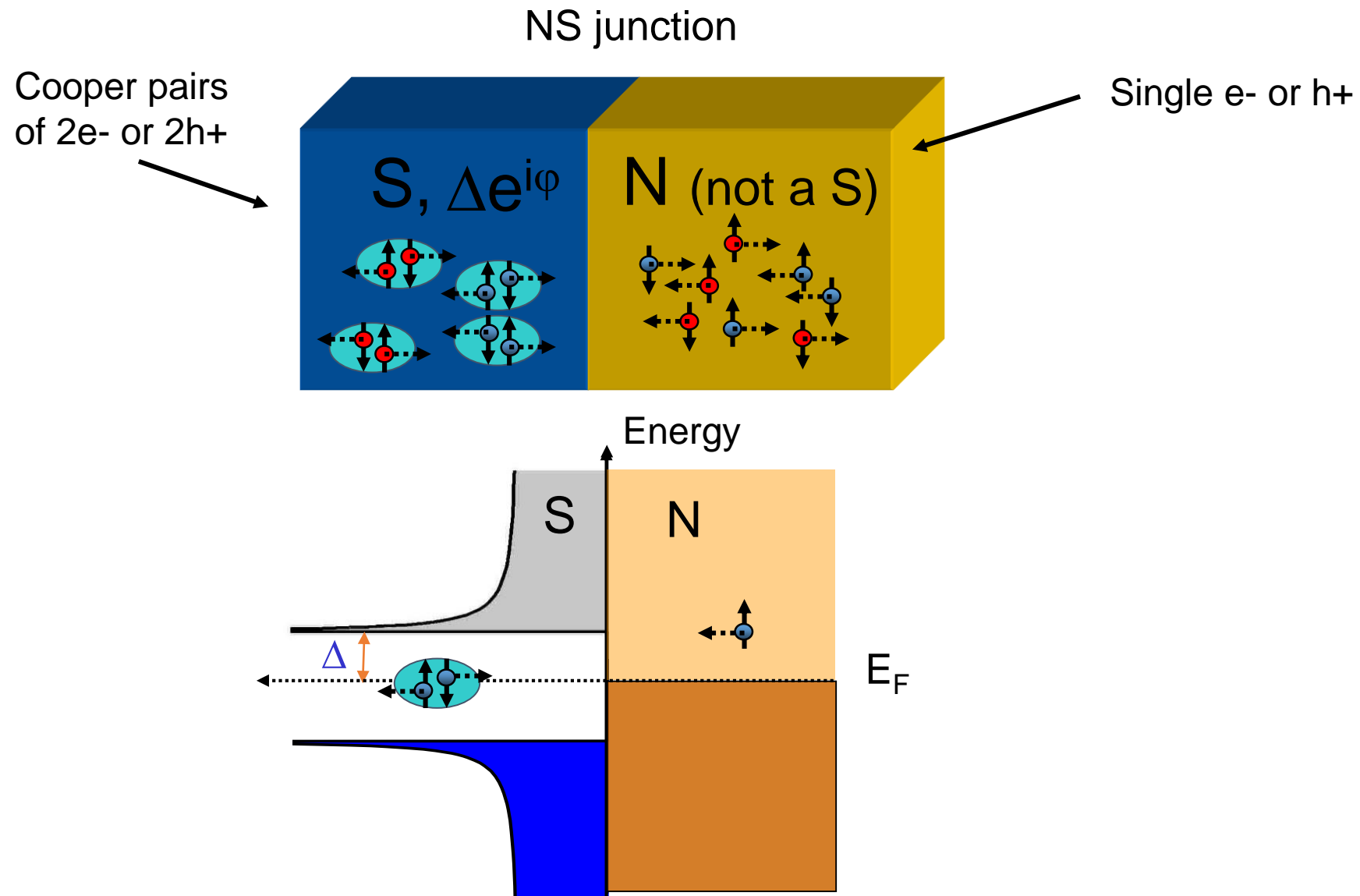
- Superconducting contacts with disordered W deposited with Ga⁺ Focused Ion Beam, after etching
=> $T_c \lesssim 5 K$, $\Delta_0 \lesssim 1 meV$ (CSNSM, Orsay)
- Larger metallic contacts with 200nm evaporated Au on top of 4nm of Ti



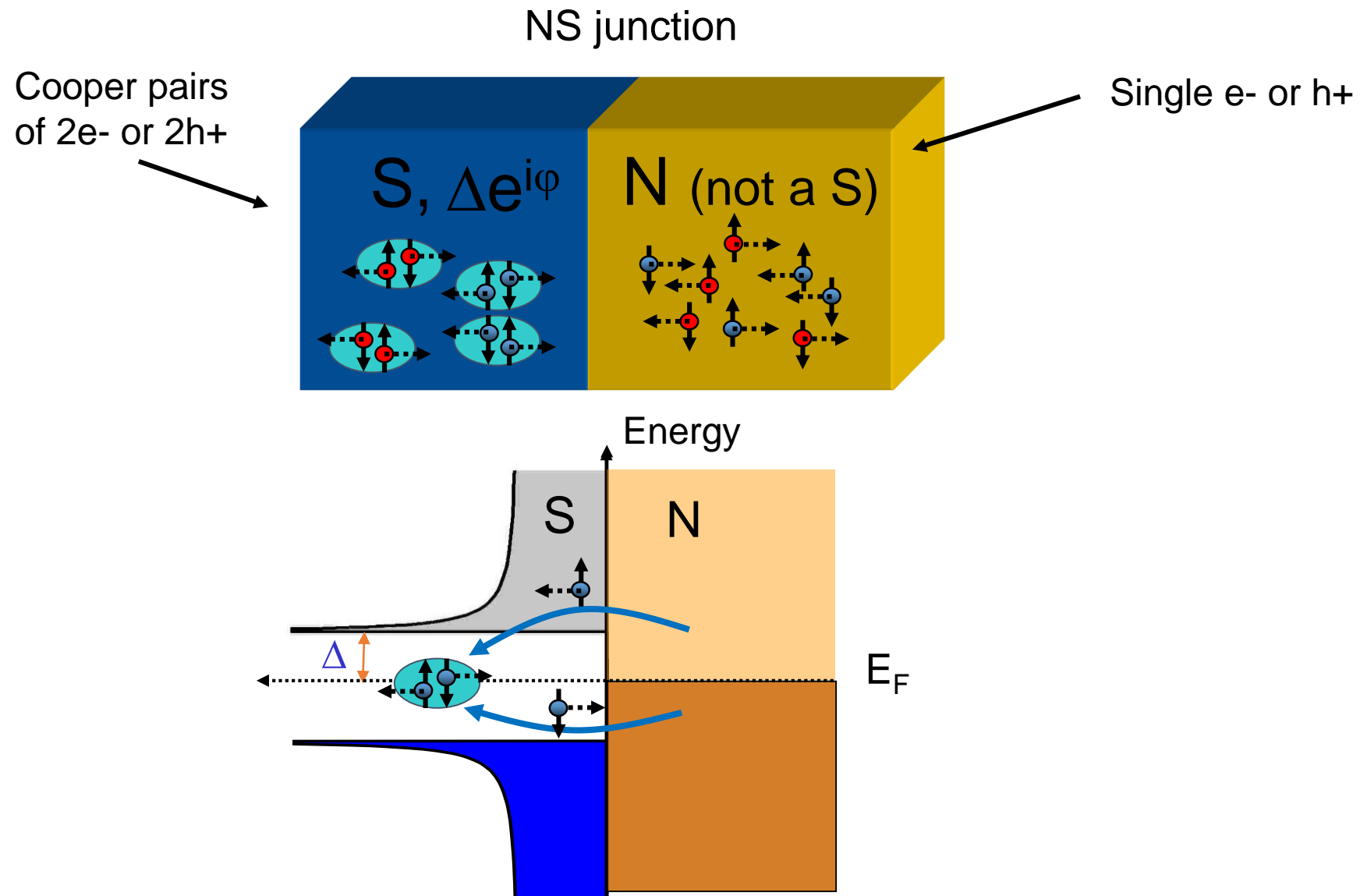
Superconducting proximity effect: Andreev reflexion



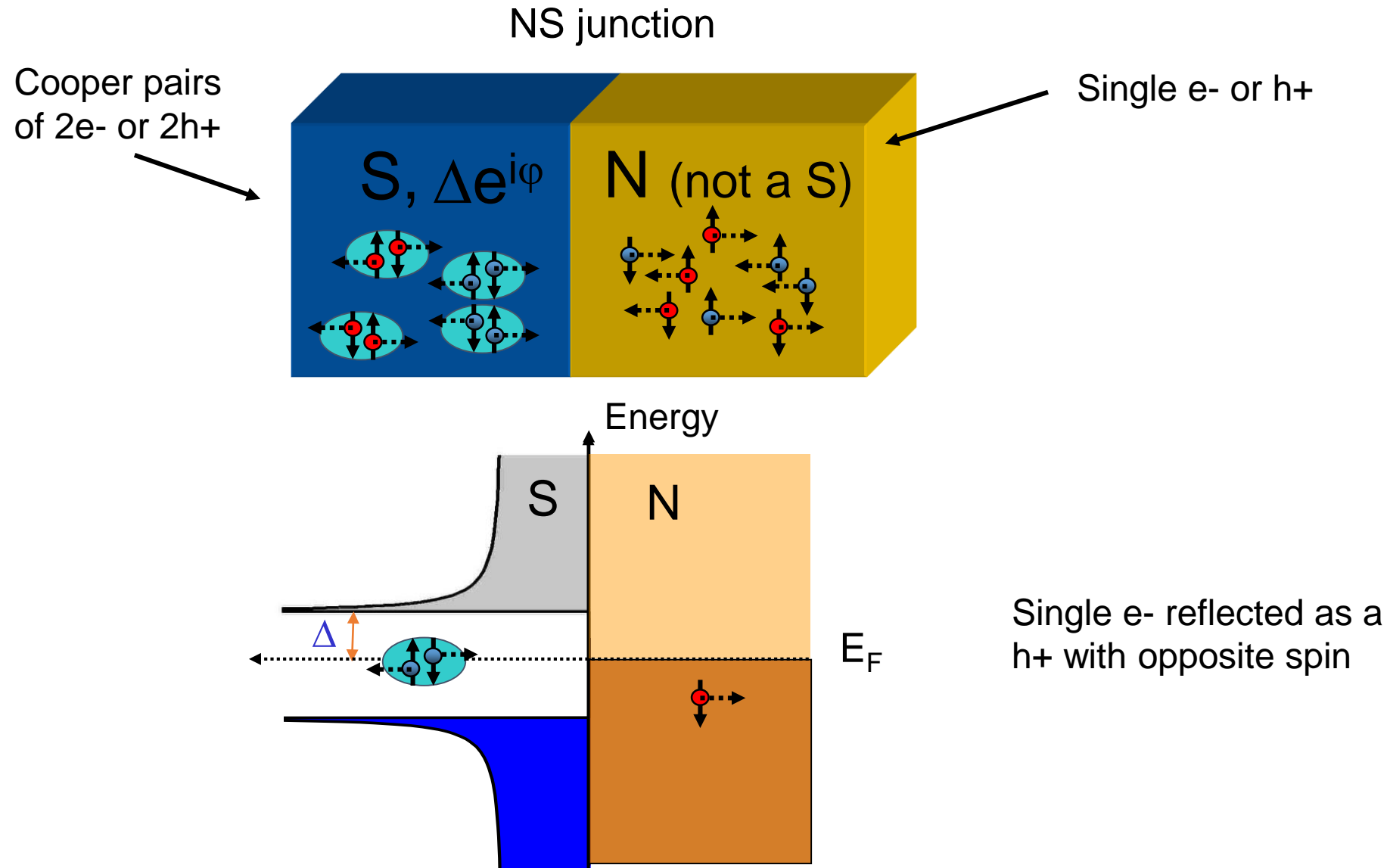
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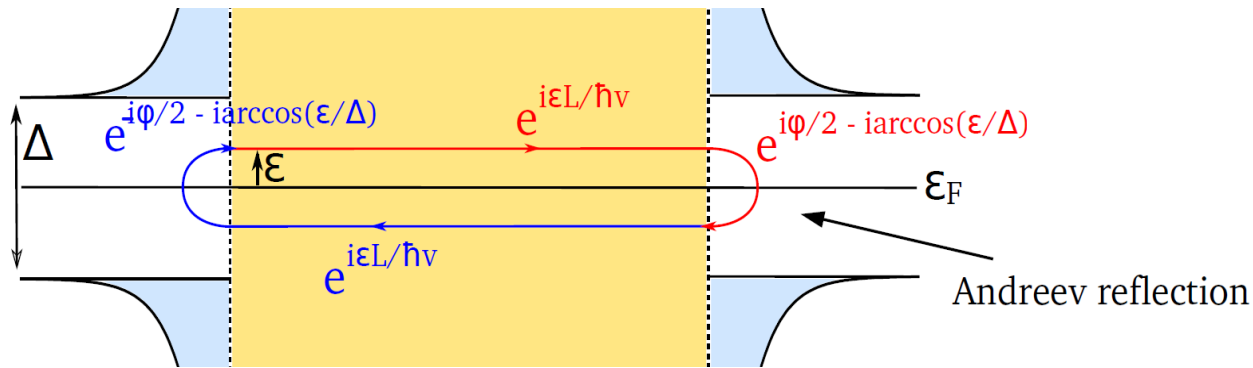


Superconducting proximity effect: Andreev reflexion



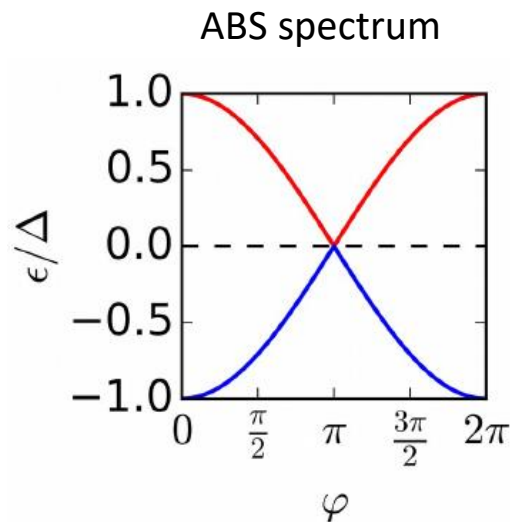
Superconducting proximity effect: Andreev Bound States

Resonance condition on accumulated phase: Andreev Bound States with eigenenergies ϵ_m

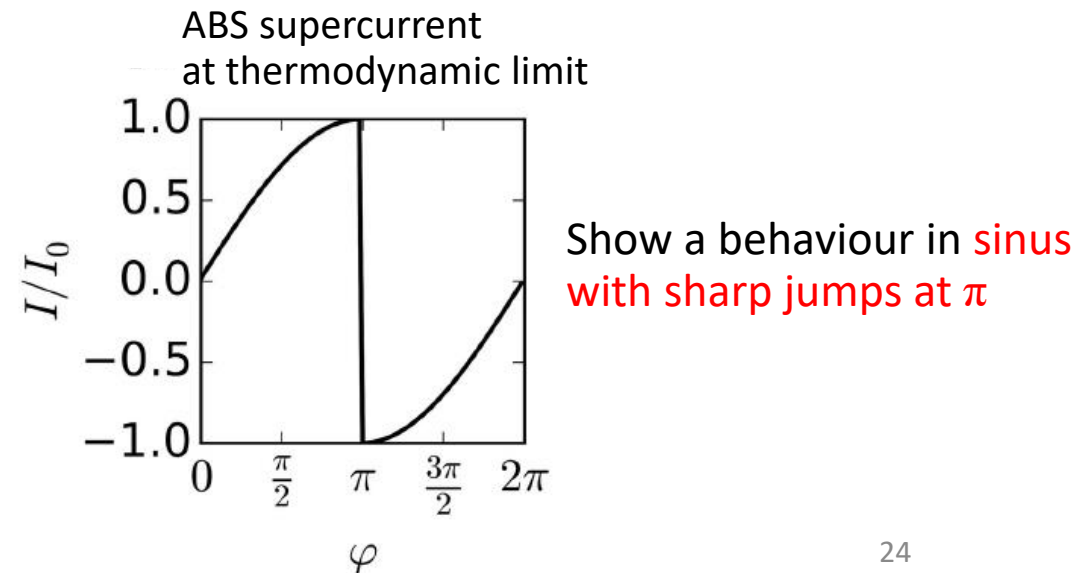


$$\cancel{\frac{2\epsilon L v}{\hbar v}} - 2 \arccos \frac{\epsilon}{\Delta_0} \pm \Delta\phi = 2\pi m$$

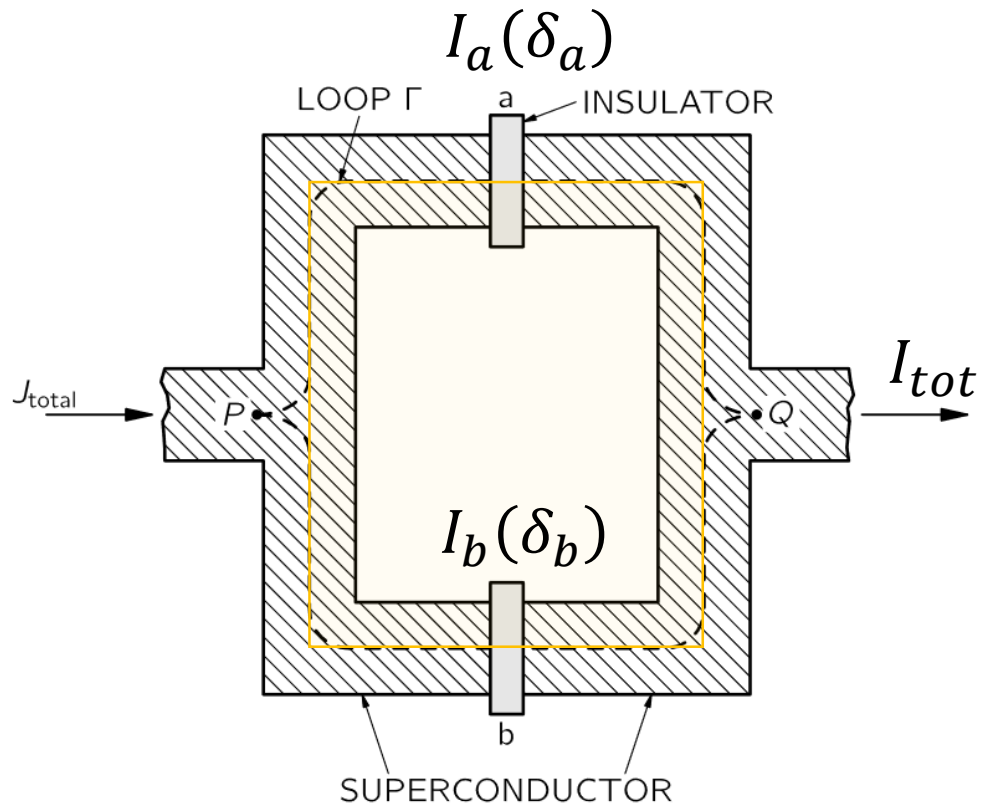
Propagation Through N Interface reflection Superconducting phase difference



$$I = \sum_{-\infty}^0 \frac{\partial \epsilon_n}{\partial \varphi} f(\epsilon_n)$$

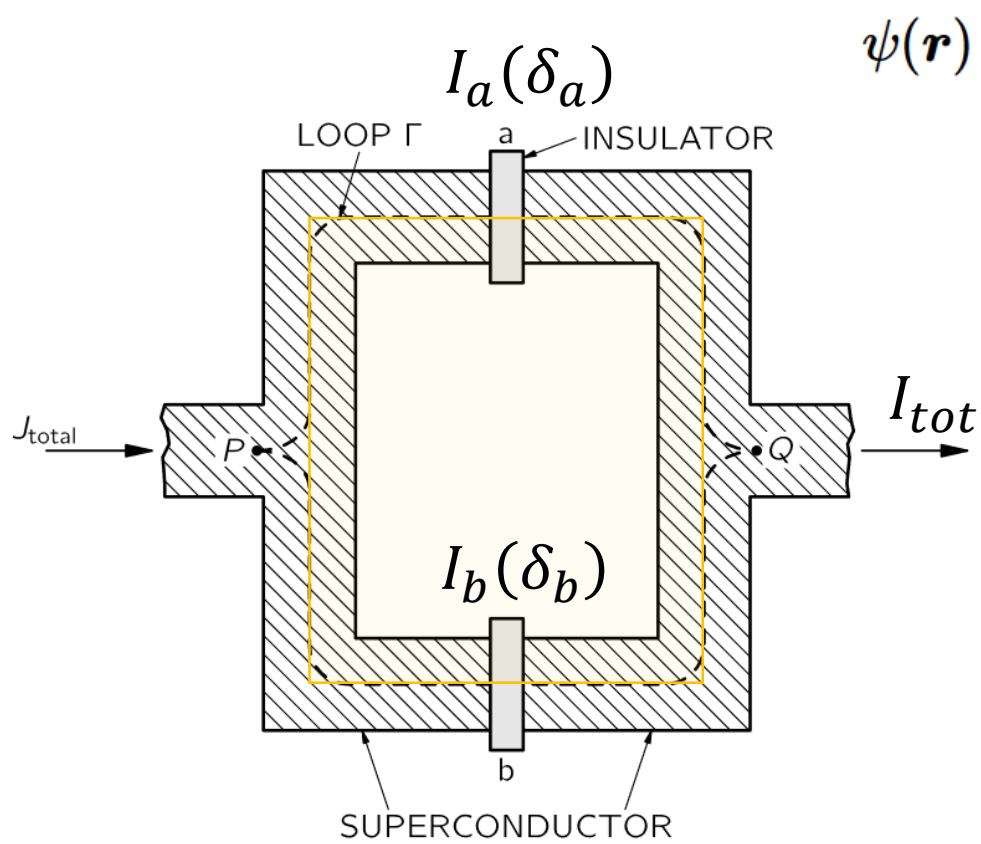


Supercurrent vs magnetic flux: SQUID



$$I_{tot}(\delta_a, \delta_b) = I_a(\delta_a) + I_b(\delta_b)$$

Supercurrent vs magnetic flux: SQUID



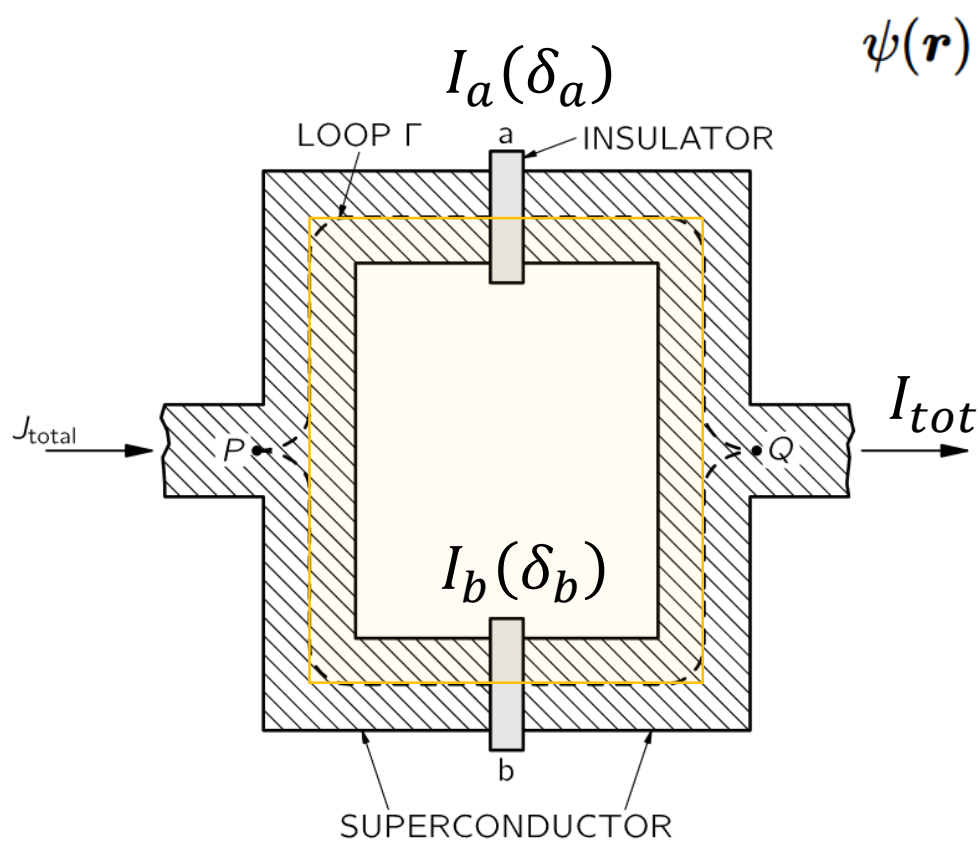
$$\psi(\mathbf{r}) = \rho^{1/2}(\mathbf{r})e^{i\theta(\mathbf{r})}$$

$$\mathbf{J} = \frac{\hbar}{m} \left(\nabla\theta - \frac{q}{\hbar} \mathbf{A} \right) \rho$$

Relate the quantum phase to the geometrical path

$$I_{tot}(\delta_a, \delta_b) = I_a(\delta_a) + I_b(\delta_b)$$

Supercurrent vs magnetic flux: SQUID



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Relate the quantum phase to the geometrical path

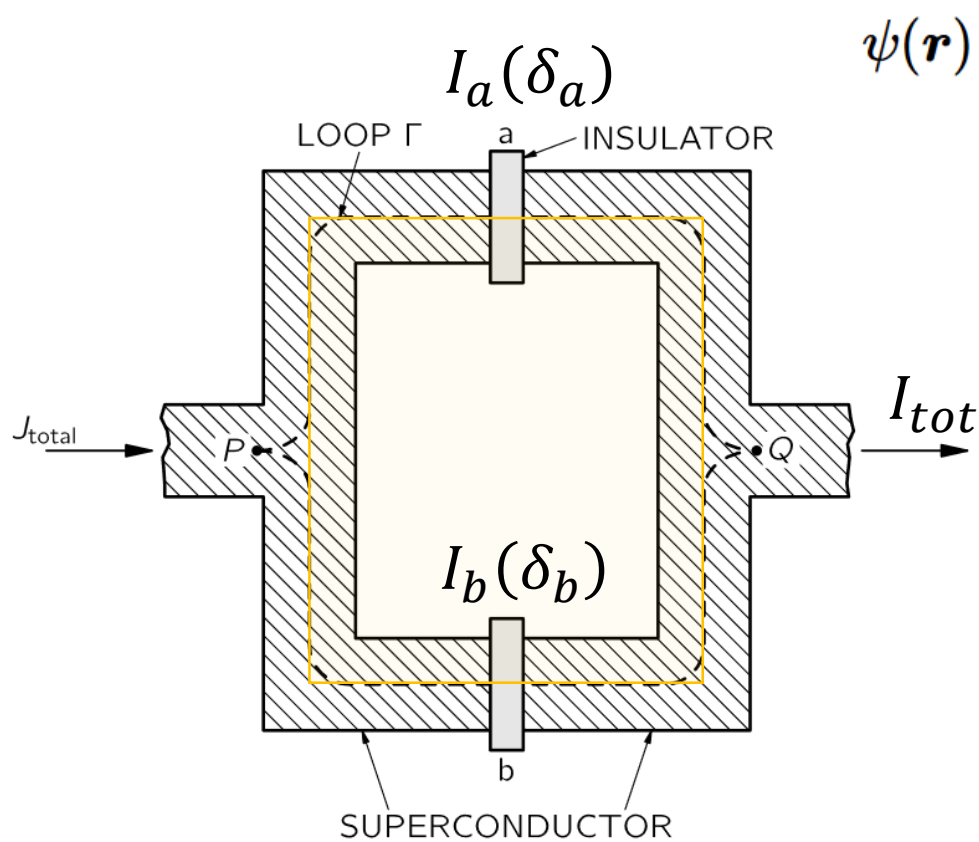
= 0 in the bulk of the superconductor

$$\delta_b - \delta_a = \frac{2q_e}{\hbar} \oint_{\Gamma} \mathbf{A} \cdot d\mathbf{s} = 2\pi \frac{B \cdot S}{\phi_0}$$

Phase difference controlled by magnetic flux (Aharonov-Bohm effect), analog to interferences in optics

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Supercurrent vs magnetic flux: SQUID



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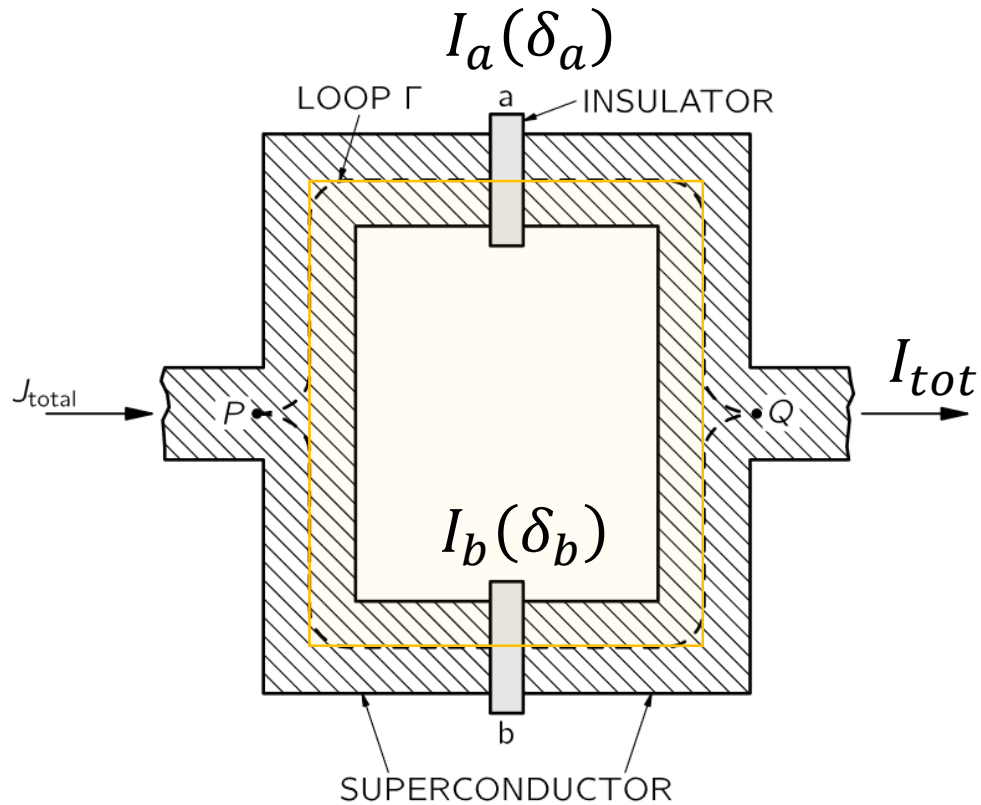
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$$I_{tot}(\delta_a, \delta_b) \rightarrow I_{tot}(\delta_a, B) = I_a(\delta_a) + I_b \left(\delta_a + 2\pi \frac{B \cdot S}{\phi_0} \right)$$

Supercurrent vs magnetic flux: SQUID

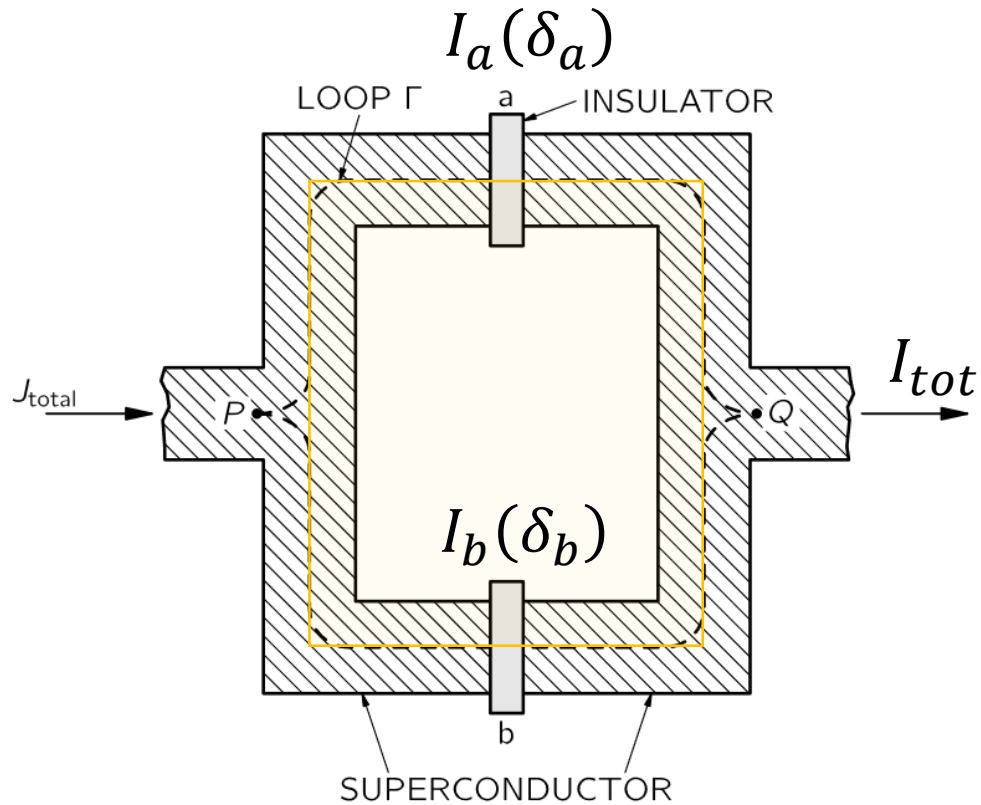


$$I_{tot}(\delta_a, B) = I_a(\delta_a) + I_b \left(\delta_b = \delta_a + 2\pi \frac{B \cdot S}{\phi_0} \right)$$

$$I_c(B) = \max_{\delta_a} I_{tot}(\delta_a, B)$$

largest current the superconducting system (symmetric SQUID) can support before becoming dissipative

Supercurrent vs magnetic flux: SQUID



$$I_{tot}(\delta_a, B) = I_a(\delta_a) + I_b \left(\delta_b = \delta_a + 2\pi \frac{B \cdot S}{\phi_0} \right)$$

$I_c(B) = \max_{\delta_a} I_{tot}(\delta_a, B)$ largest current the superconducting system (symmetric SQUID) can support before becoming dissipative

$$I_c \left(B + n \frac{\phi_0}{S} \right) = I_c(B)$$

$$\Delta B = \frac{\phi_0}{S}$$

Current of the whole junctions ΔB periodic

Supercurrent vs magnetic flux: many paths

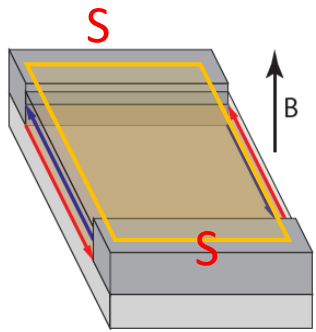
Aharonov-Bohm effect dephasing
with B



time-of-flight dephasing
in optics

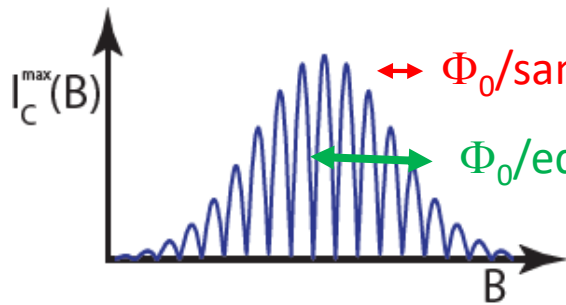
Supercurrent interferences can distinguish transport geometries

2D TI



Only 2 identical
paths (edges)

$$\Delta B = \frac{\phi_0}{S}$$

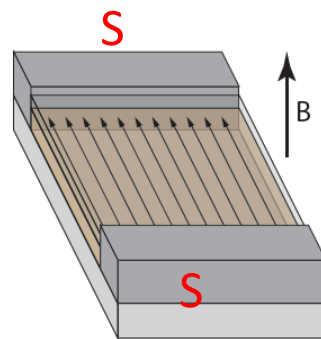


« Young pattern »

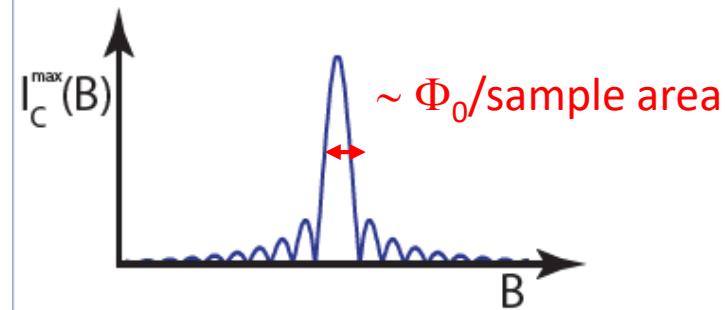
$$\text{FT}(\text{narrowGate} * 2\text{Dirac}) = \text{FT}(\text{narrowGate}) * \text{FT}(2\text{Dirac})$$

SQUID-like behaviour in a single wire

Metal
 $L \ll W$



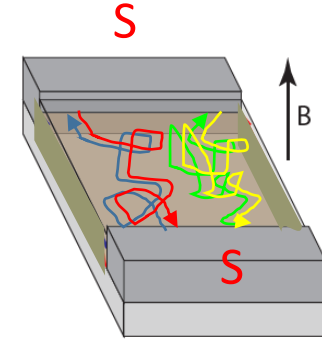
Many identical
ballistic paths



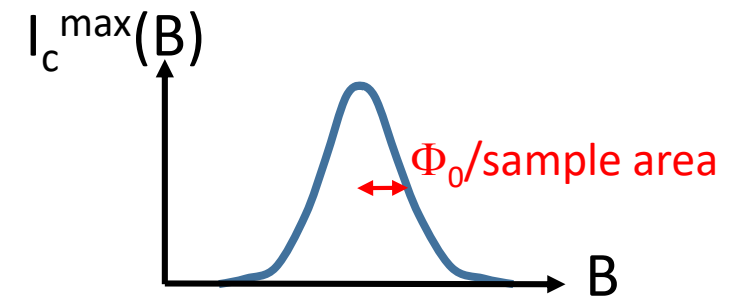
« Fraunhofer pattern »

$$\text{FT}(\text{wideGate})$$

Metal
 $L \gg W$



Many diffusive
paths

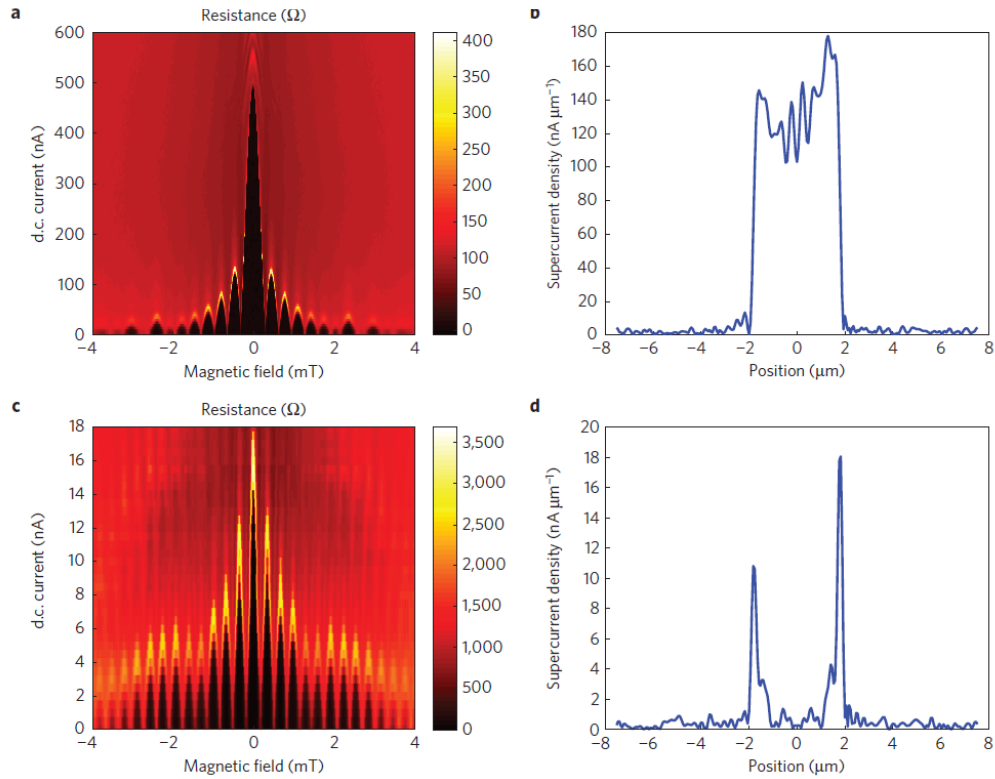
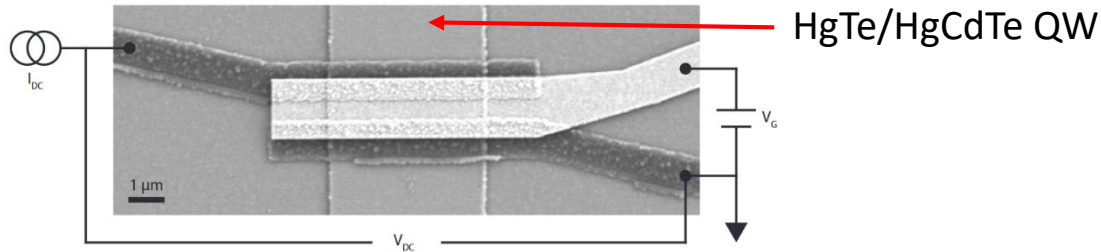


Gaussian decay

$$\text{FT}(\text{narrowGate})$$

Critical current vs flux: experiments

$L \ll W$ case, from metal to 2D TI

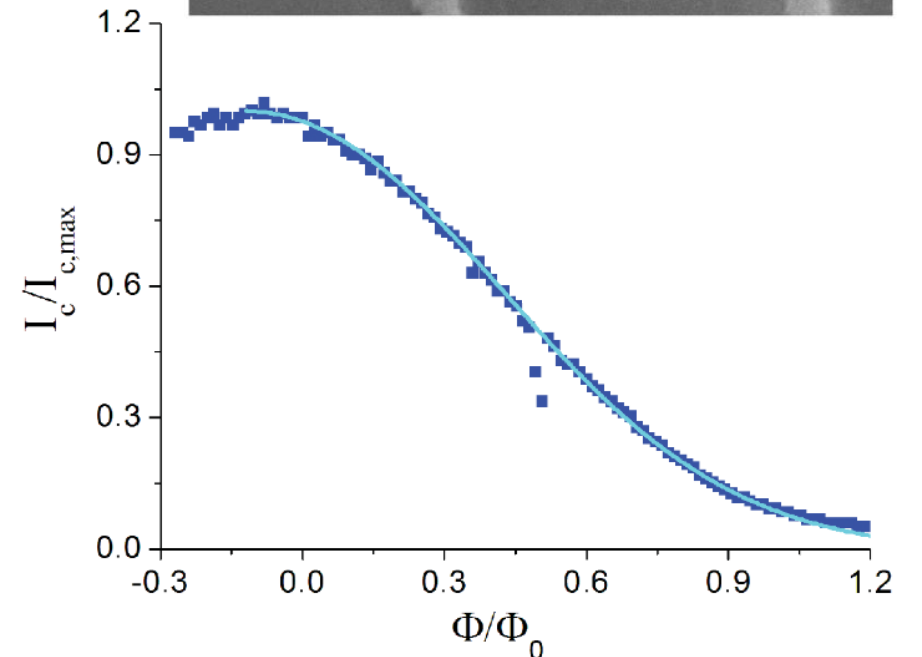
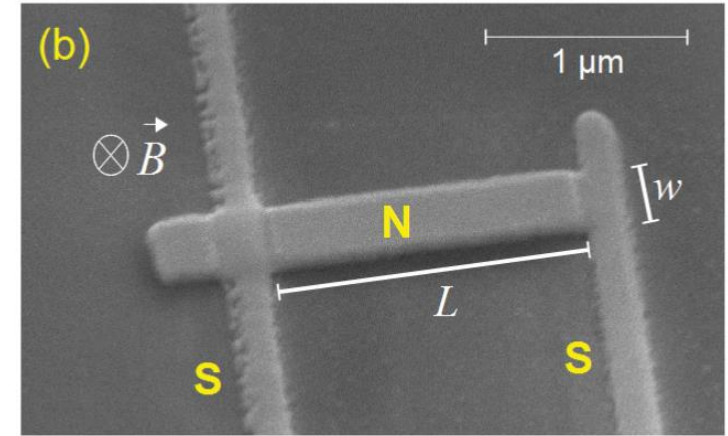


Metallic phase
 $V_g = 1.05V$

2D TI phase
 $V_g = -0.42V$

Hart et al., Nat. Phys. 2014

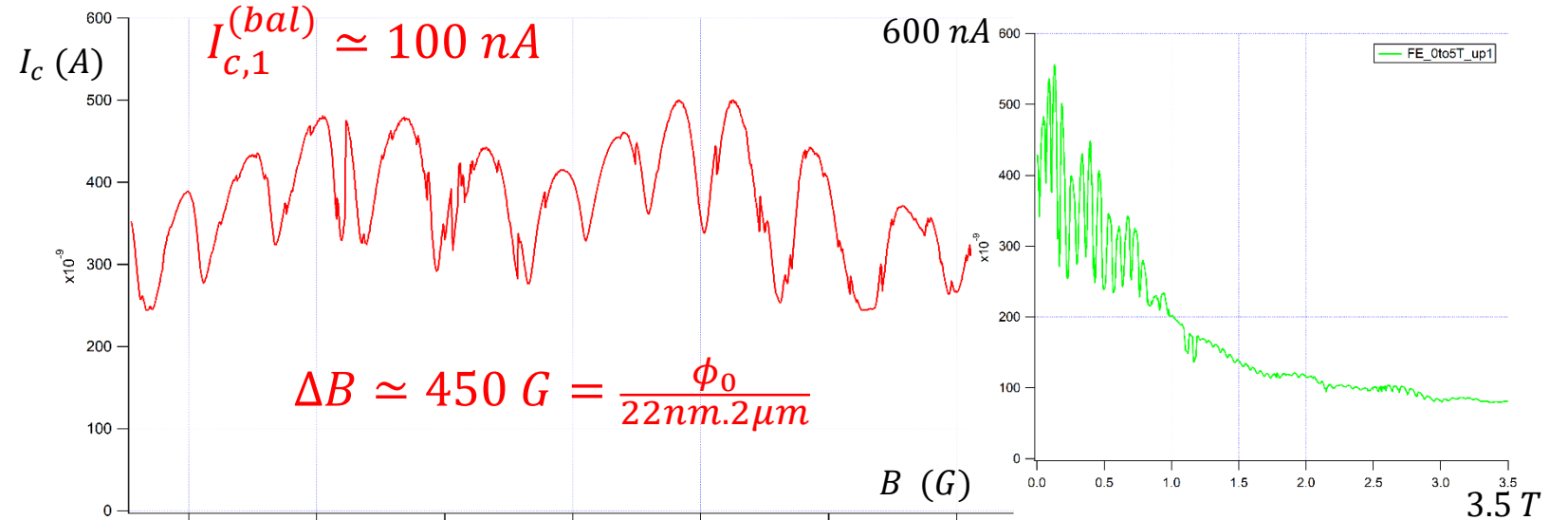
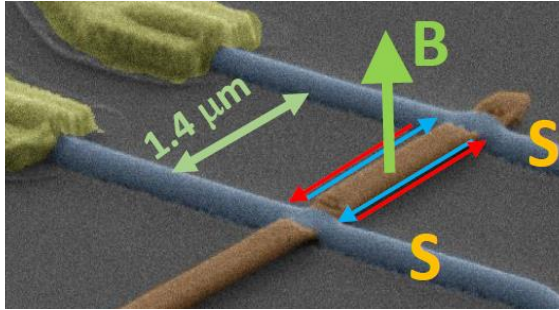
$L \gg W$ metallic case



Chiodi et al., PRB 2012 ³²

Critical current vs flux: experiments on Bi

Ongoing experiment, ~100nm large Bi wire, 111 axis

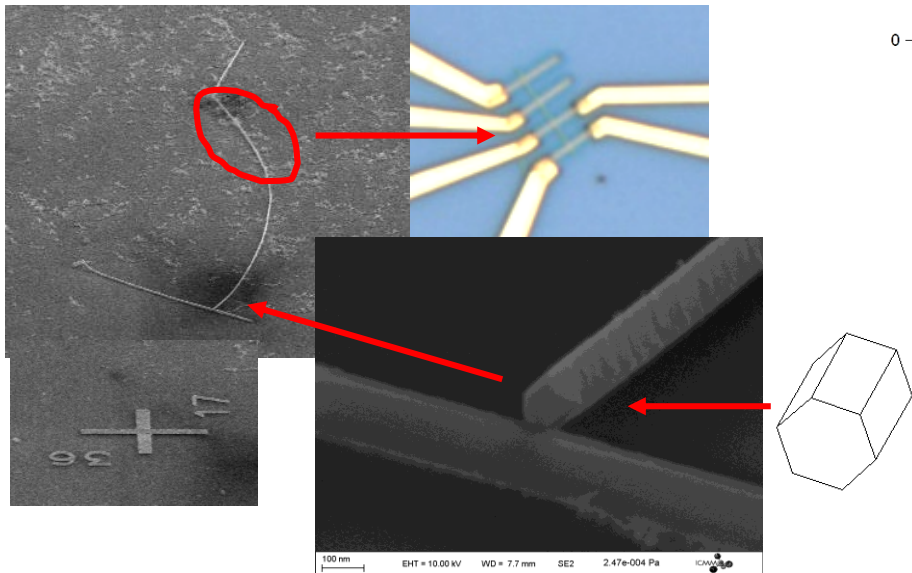


SQUID-like behaviour
in a single wire

$$\Delta B \approx 1 \text{ T} = \frac{\Phi_0}{1 \text{ nm} \cdot 2 \mu\text{m}}$$

Survives at very
high magnetic field

=> Current carried by a small number of narrow paths



For previous experiments on Bi:
Murani et al., Nat. Comm. 2017

1D hinge states really topologically protected ?

Critical current vs flux measurement sensitive to the conduction channels geometry and distribution.

Signature of topology other than the interference between a few channels at the edges of the sample ?

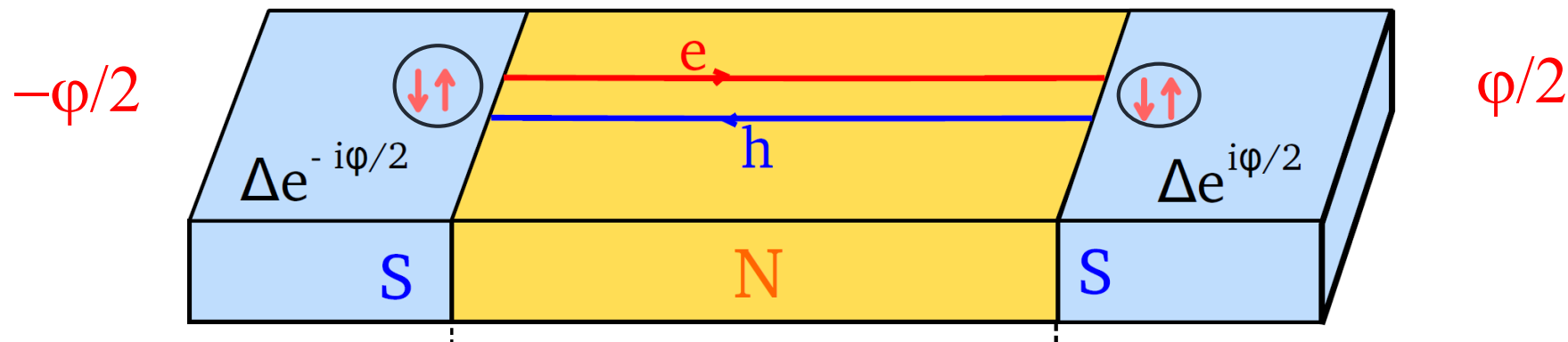
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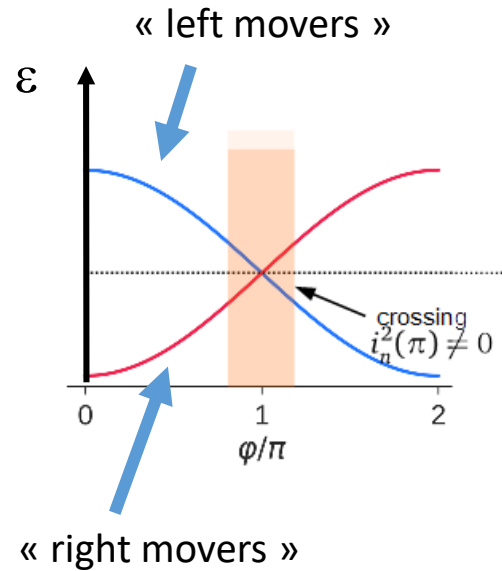
Supercurrent vs phase

What is the behaviour of the S-Bi-S junction when we impose a phase bias ?



Supercurrent vs phase: topo vs trivial

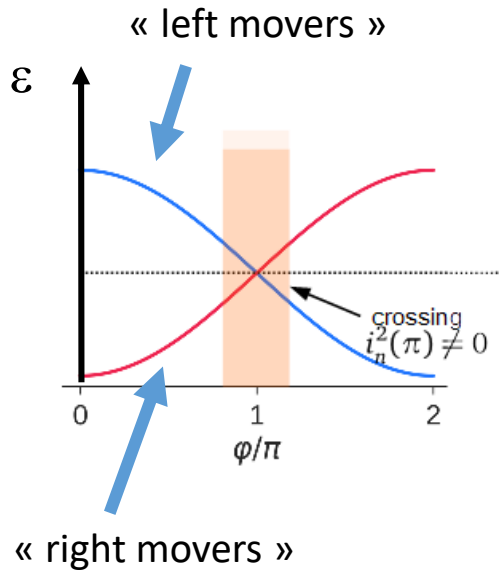
What happens when there is scattering ?



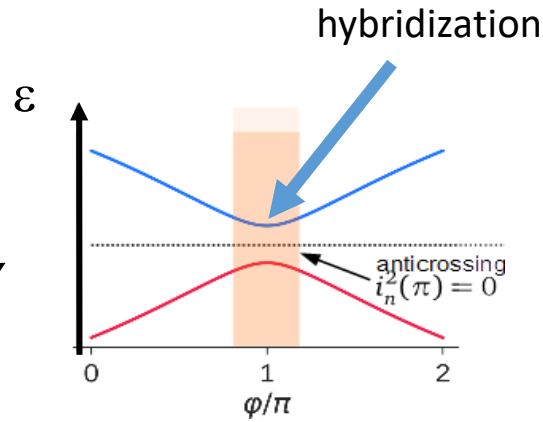
Supercurrent vs phase: topo vs trivial

What happens when there is scattering ?

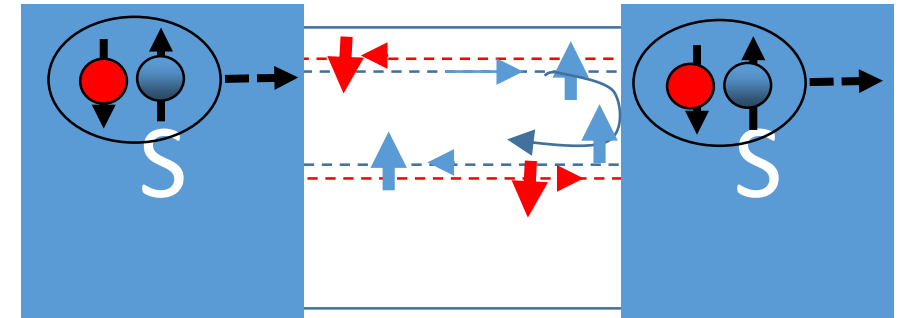
Kwon et al., Eur. Phys. J. B 2004



Trivial case



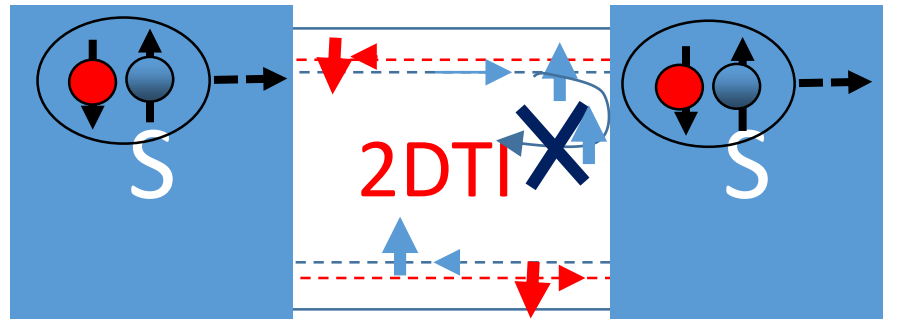
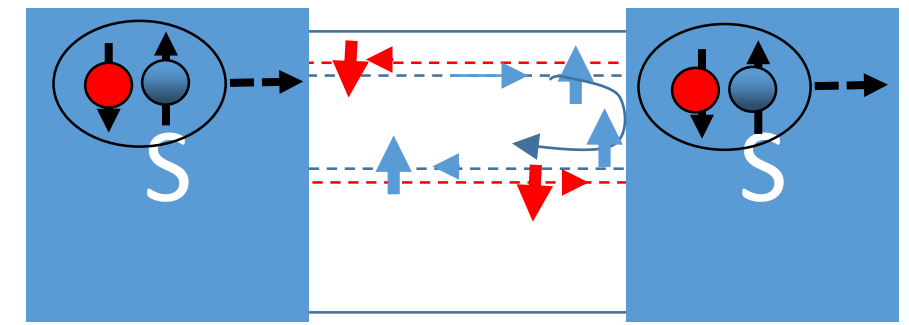
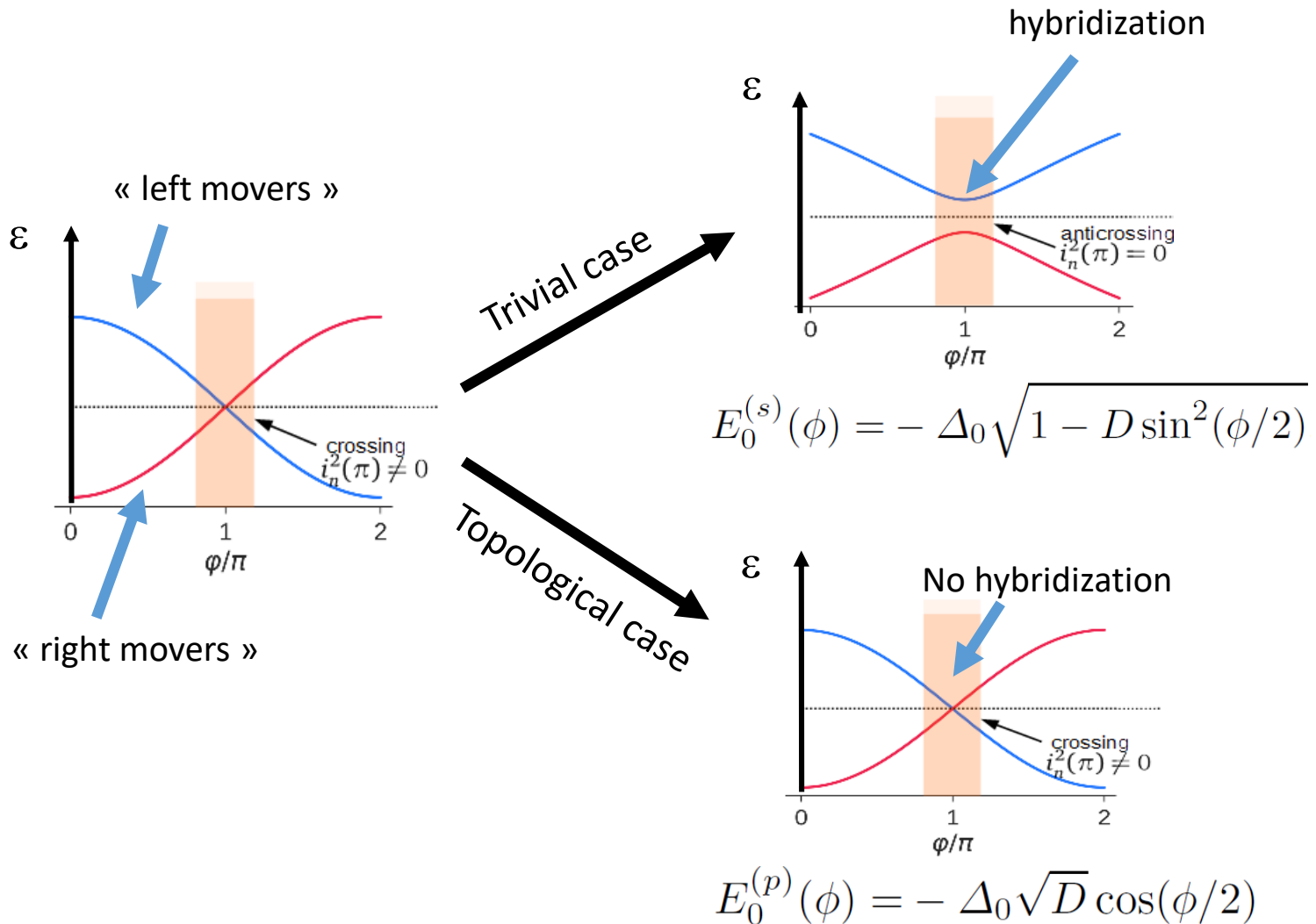
$$E_0^{(s)}(\phi) = -\Delta_0 \sqrt{1 - D \sin^2(\phi/2)}$$



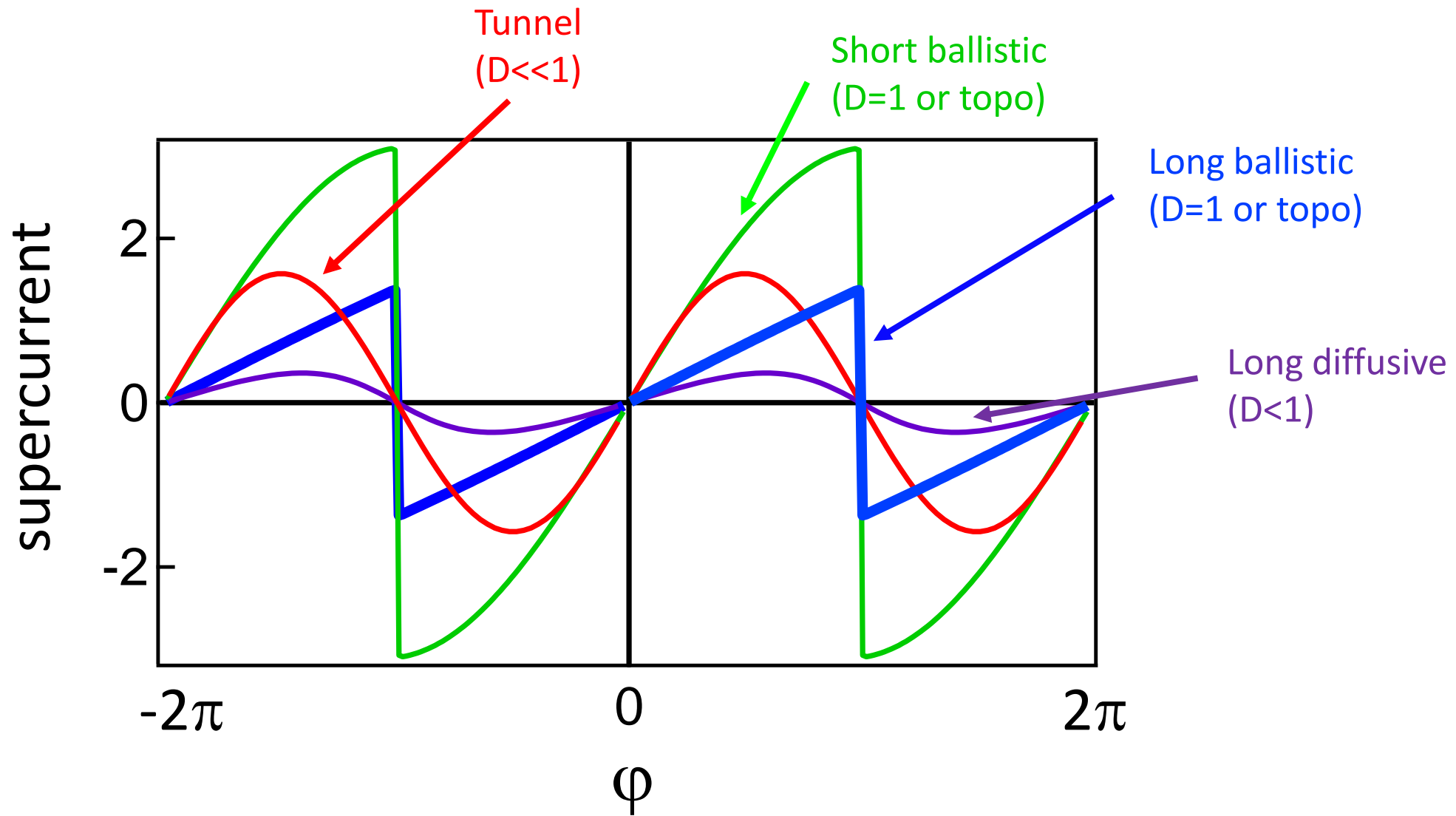
Supercurrent vs phase: topo vs trivial

What happens when there is scattering ?

Kwon et al., Eur. Phys. J. B 2004



Supercurrent vs phase: expectations

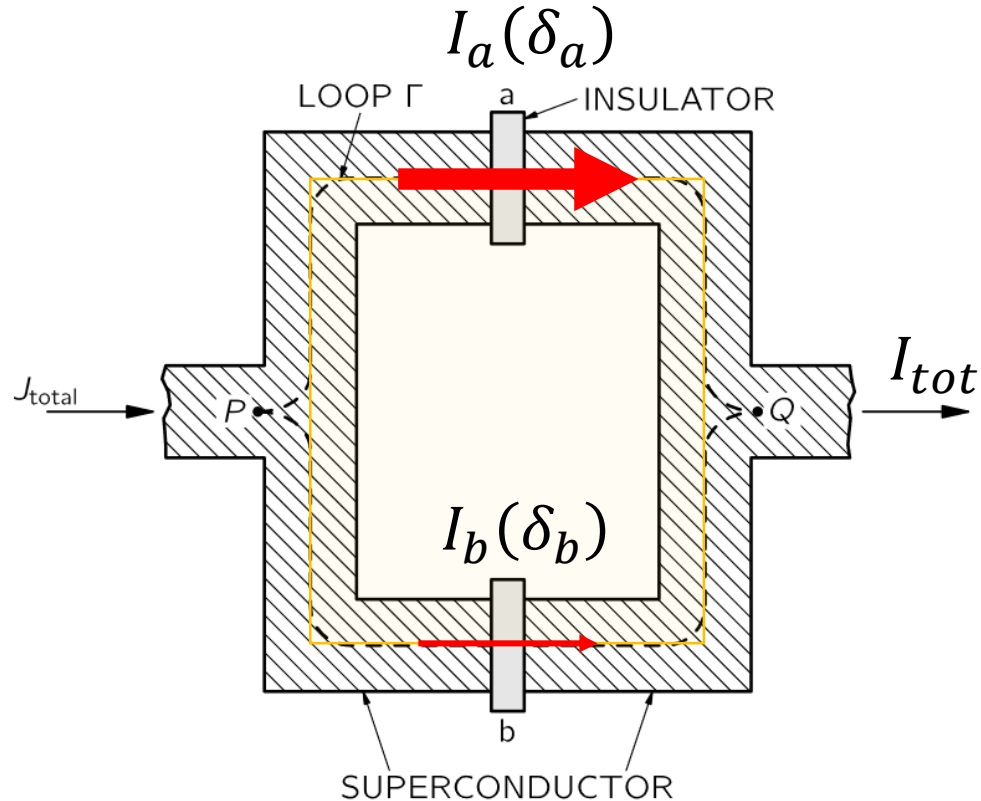


Supercurrent vs phase: measurement with an asymmetric SQUID

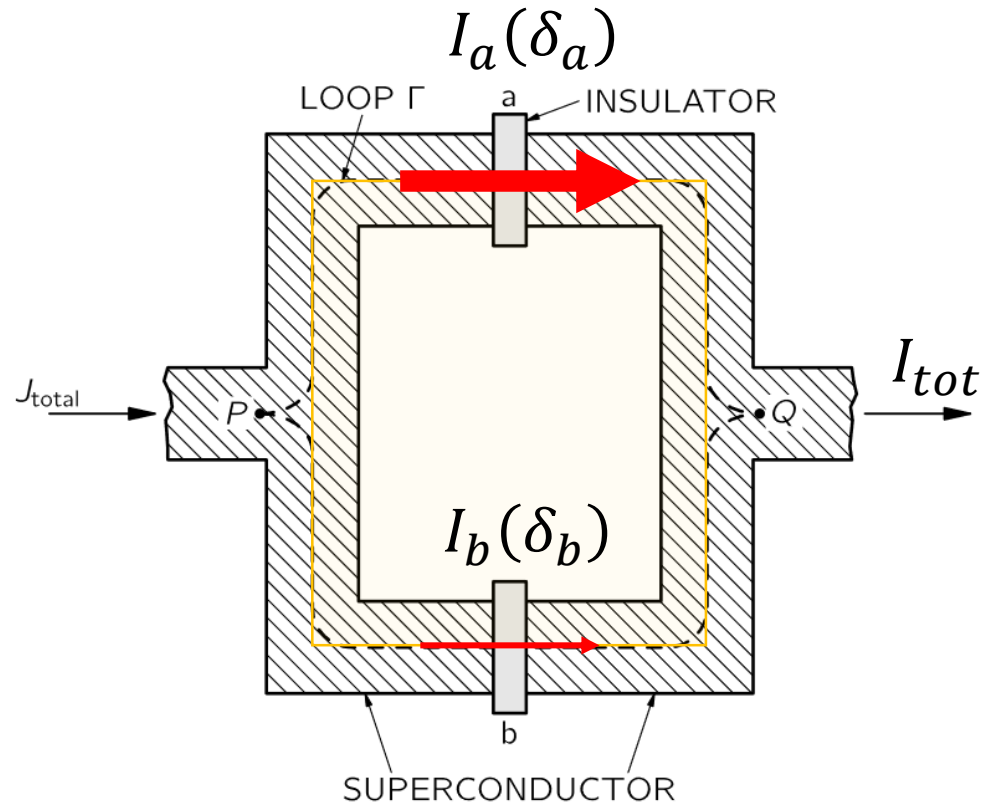
What happens when $\max I_a(\delta_a) \gg \max I_b(\delta_b)$?

Supports the largest current when $\delta_a = \theta$ is such that
the current in I_a is maximum

$$I_c(B) = \max_{\delta_a} I_{tot}(\delta_a, B) \simeq I_{tot}(\theta, B)$$



Supercurrent vs phase: measurement with an asymmetric SQUID



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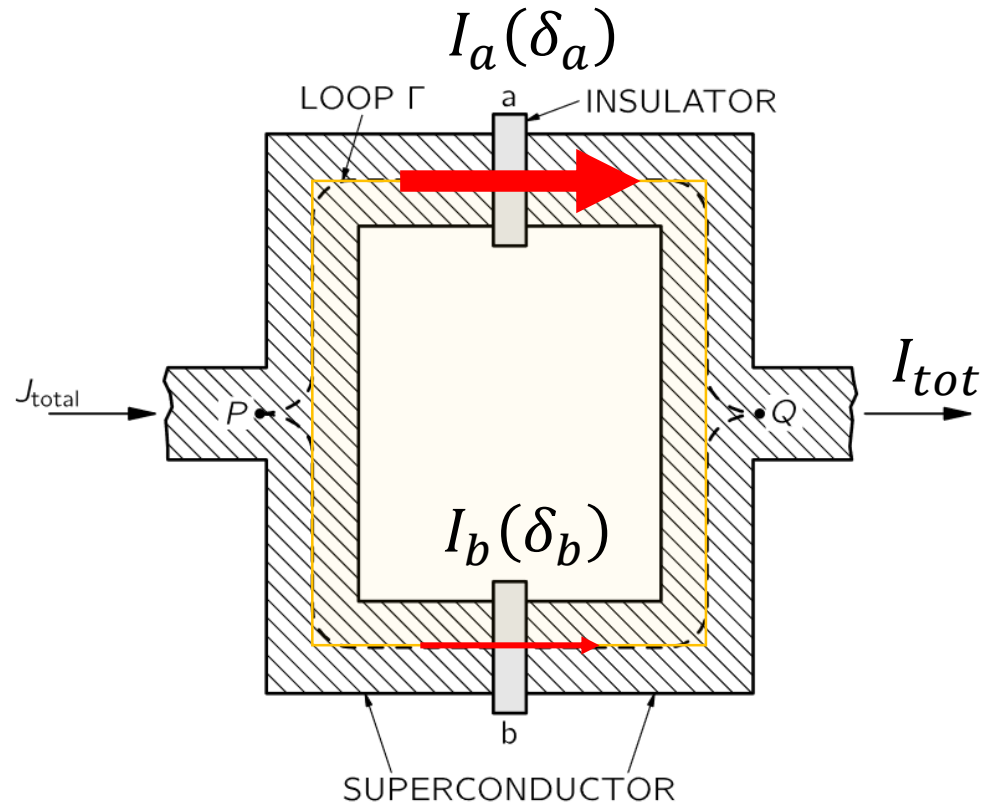
Supports the largest current when $\delta_a = \theta$ is such that the current in I_a is maximum

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$$I_c(B) \simeq I_{tot}(\theta, B) = I_a(\theta) + I_b\left(\delta_b = \theta + 2\pi\frac{B.S}{\phi_0}\right)$$

The big junction imposes $\delta_a = \theta$
and B imposes $\delta_b - \delta_a = 2\pi\frac{B.S}{\phi_0}$

Supercurrent vs phase: measurement with an asymmetric SQUID



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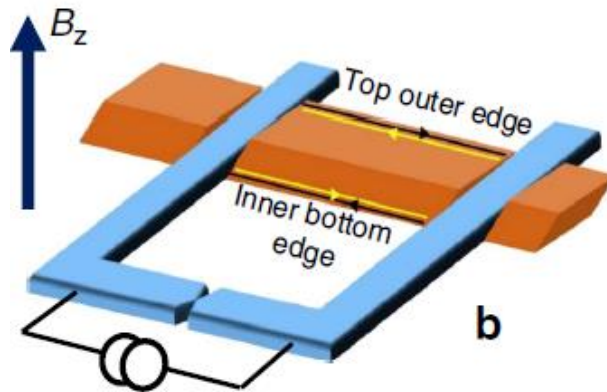
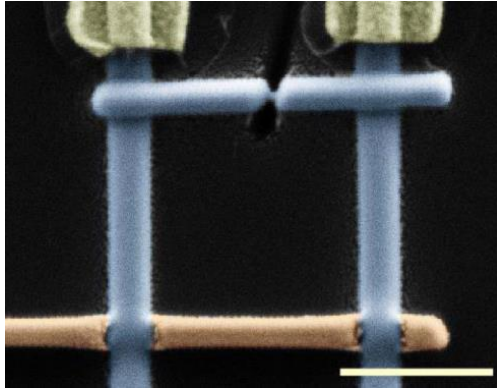
$$I_c(B) \simeq I_{tot}(\theta, B) = I_a(\theta) + I_b\left(\delta_b = \theta + 2\pi\frac{B \cdot S}{\phi_0}\right)$$

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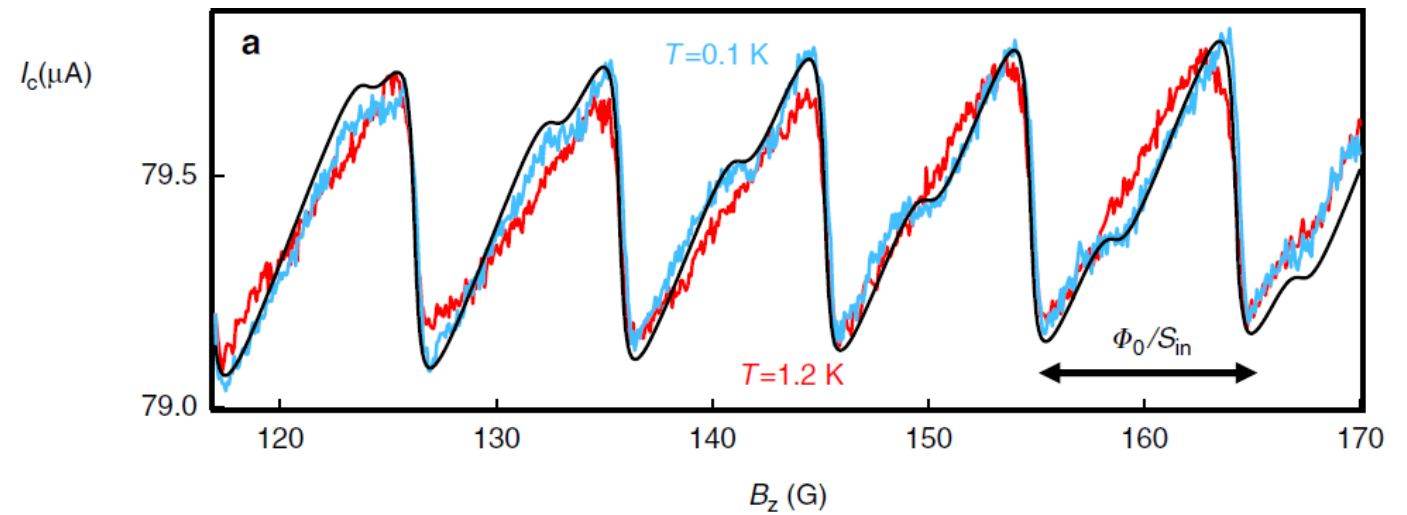
=> can measure supercurrent vs phase $I_b(\varphi)$ with $I_c(B)$

$$I_c\left(B = \varphi \cdot \frac{\phi_0}{2\pi S}\right) \simeq I_a(\theta) + I_b(\theta + \varphi)$$

Supercurrent vs phase: previous experiments



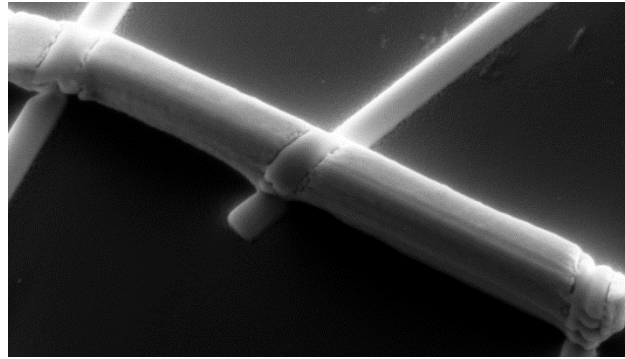
Murani et al., Nat. Comm. 2017



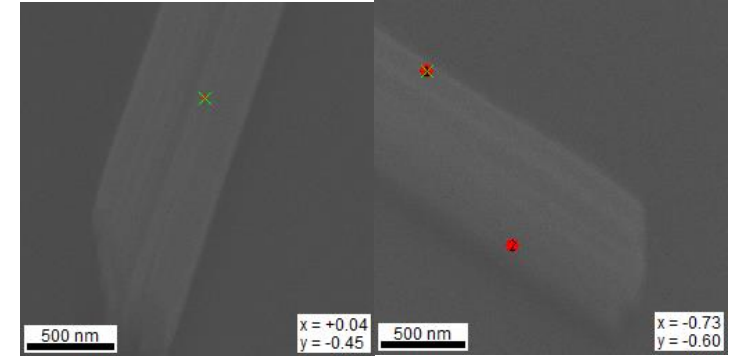
THANK YOU

Open questions

Can a big supercurrent be supported by many hinge states arranged in steps ?



R. S. Deacon
Ishibashi's group, RIKEN



What is the effect of defects on the surface of Bi ?
What about strain ?
Revealing topological nature with screw dislocations ?
(Nayak et al., Cond. Mat. 2019)

What happens at high magnetic field ?
(Queiroz and Stern, Cond. Mat. 2019)

